Stationary Binary Inspiral

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BACKGROUND AND MOTIVATION

- LIGO, LISA and strong grav waves
- NumRel not likely to handle many orbits by itself, but can/will handle plunge.
- Idea: a complementary approximation that is appropriate while orbit is changing slowly
- The periodic standing-wave approximation: exact, helically symmetric solution to Einstein's equations
- Provides a quasistationary sequence before plunge, and initial value solution for plunge

OVERVIEW

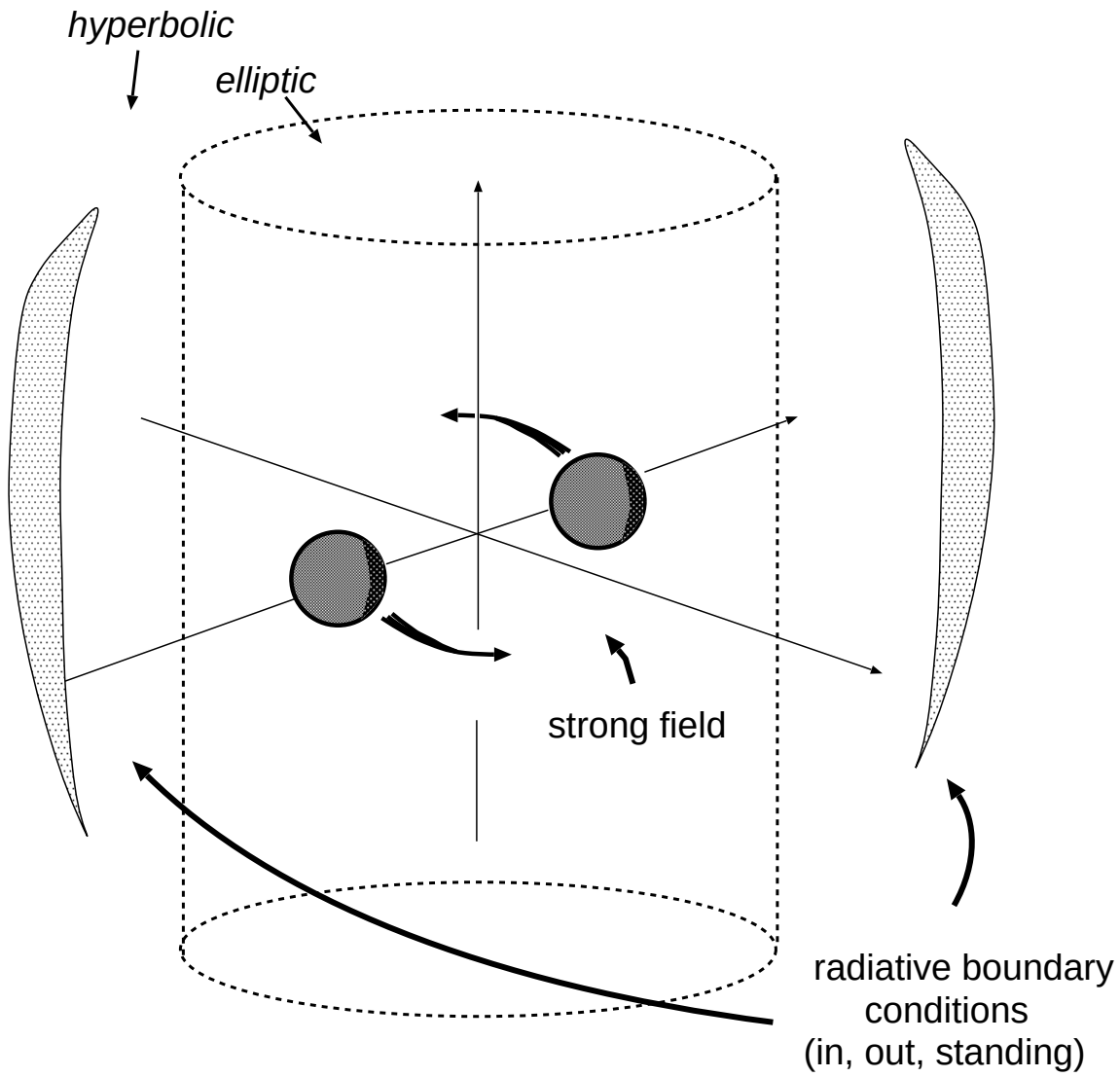
- Not asymptotically flat in usual sense
- Hyperbolic Cauchy 4D prob $\longrightarrow$ mixed BV 3D prob
- Mixed-nature makes relaxation and multigrid difficult or impossible
- Different methods of solution being considered: FDM, FDM+FEM, spectral.
- Meaning of "standing wave" not clear. Meaning tightly tied to method.
- Many new issues to deal with:
  - Mixed PDEs, fundamental issues
  - Mixed PDEs, numerical implementation
  - PSW as an approximation to outgoing

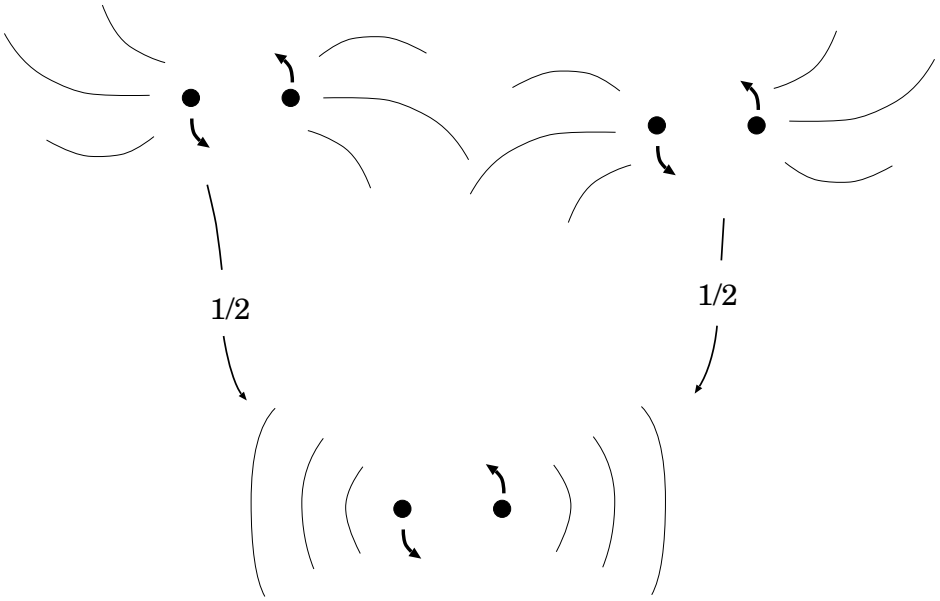
  Toy models useful for majority of them. So far scalar nonlinear 3D and linearized gravity.

Scalar Model

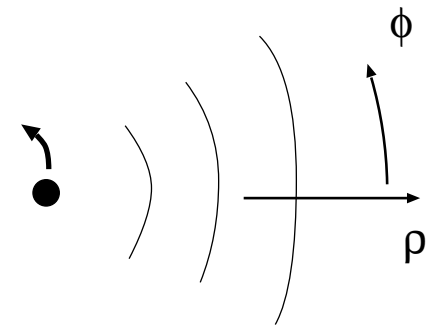
- $\nabla^2\psi - \partial_t^2\psi + F(r,\theta,\psi)$
  $= \text{Source}$
- $\text{Source} = \text{Source}(r,\theta,\overbrace{\phi - \Omega t}^{\varphi})$
- $\psi = \psi(r,\theta,\varphi)$ and

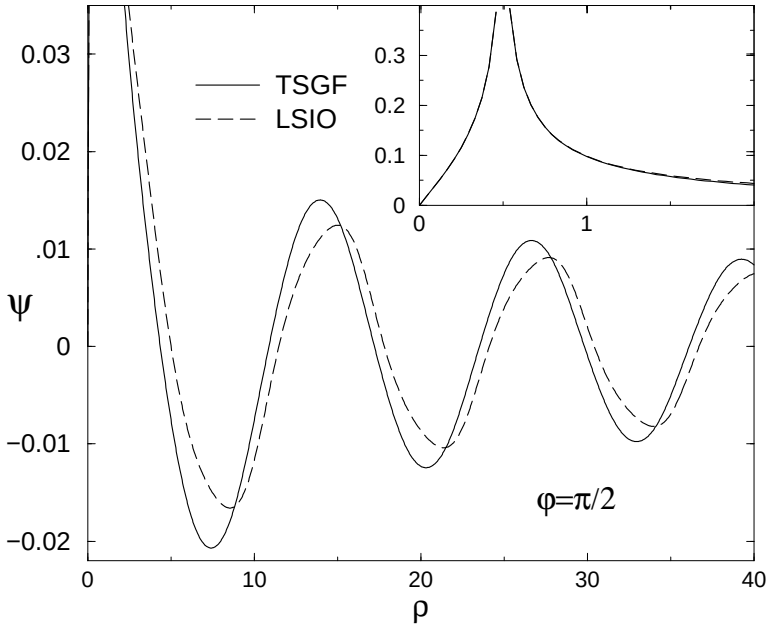
$$\partial_r^2\psi + \left(\frac{1}{r^2\sin^2\theta} - \Omega^2\right)\partial_\varphi^2 + \frac{1}{r^2}\partial_\theta^2\psi$$
$$+\frac{1}{r^2}\cot\theta\ \partial_\theta\psi + \frac{1}{r}\partial_r\psi + F(r,\theta,\psi)$$
$$= \text{Source}$$

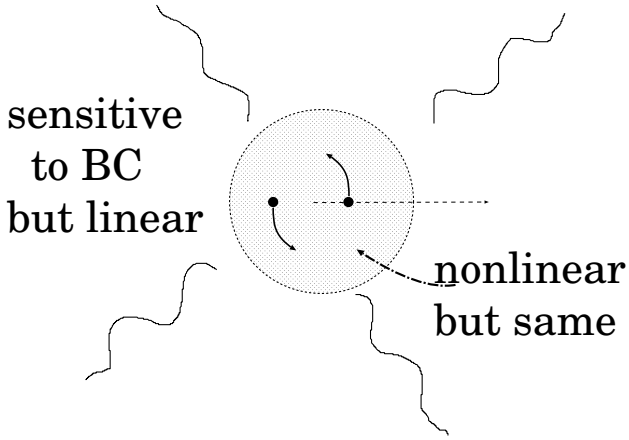

hyperbolic
elliptic
strong field
radiative boundary
conditions
(in, out, standing)



For better pictures, and movies, see
`http://www.physics.utah.edu/~rachel`


ϕ
ρ


TSGF
LSIO
ψ
ρ
φ=π/2
0.03
0.02
.01
0
−0.01
−0.02
0
10
20
30
40
0.3
0.2
0.1
0
0
1


sensitive
to BC
but linear
nonlinear
but same

STANDING WAVE "BOUNDARY CONDITIONS"

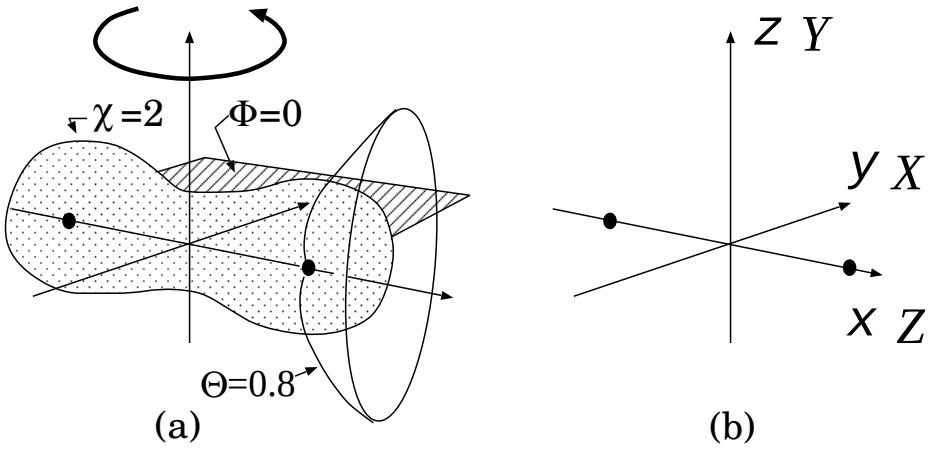
- In linear theory: half out + half in

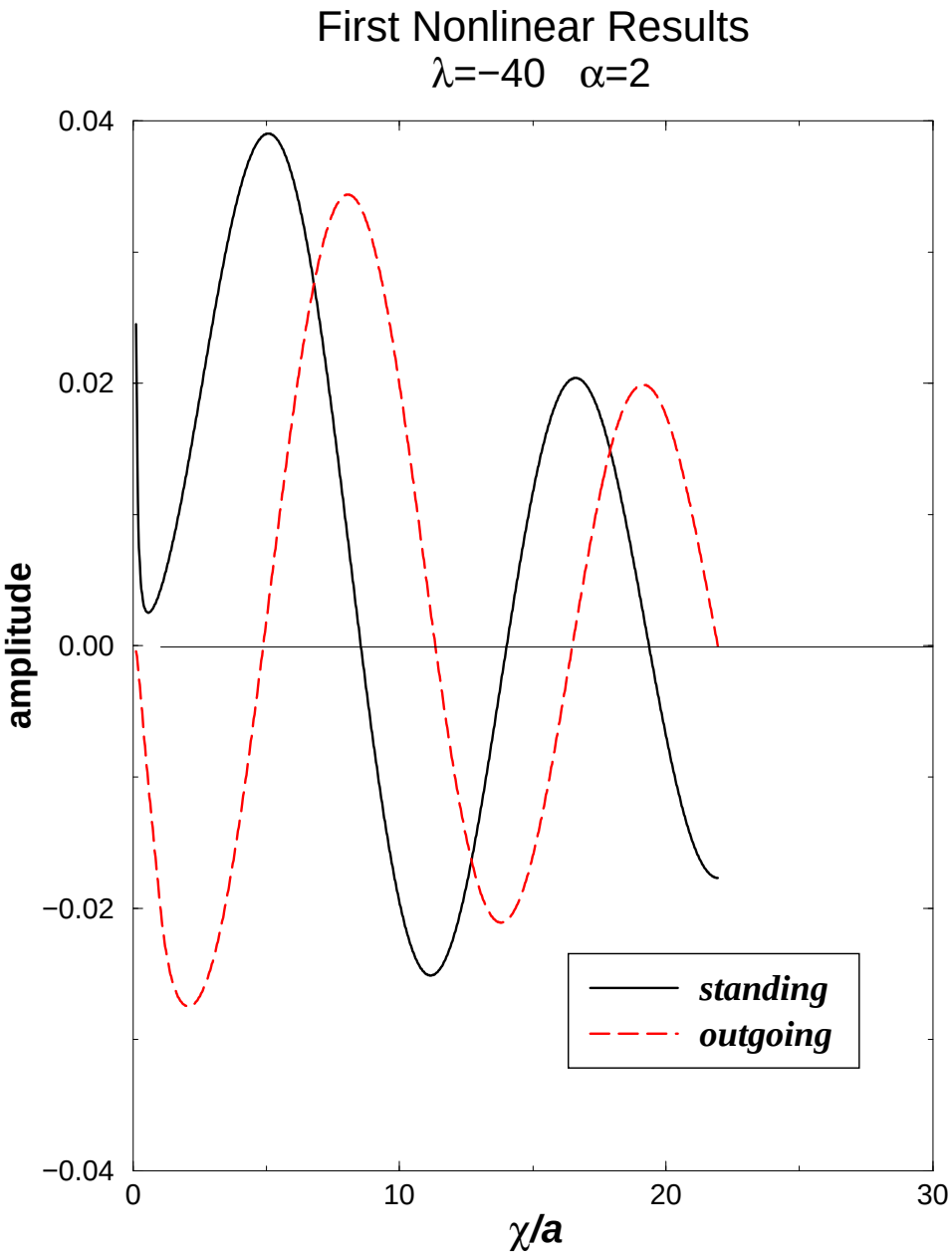
- Can't use in GR since no "out" or "in" and no superposition. We use two very different definitions.

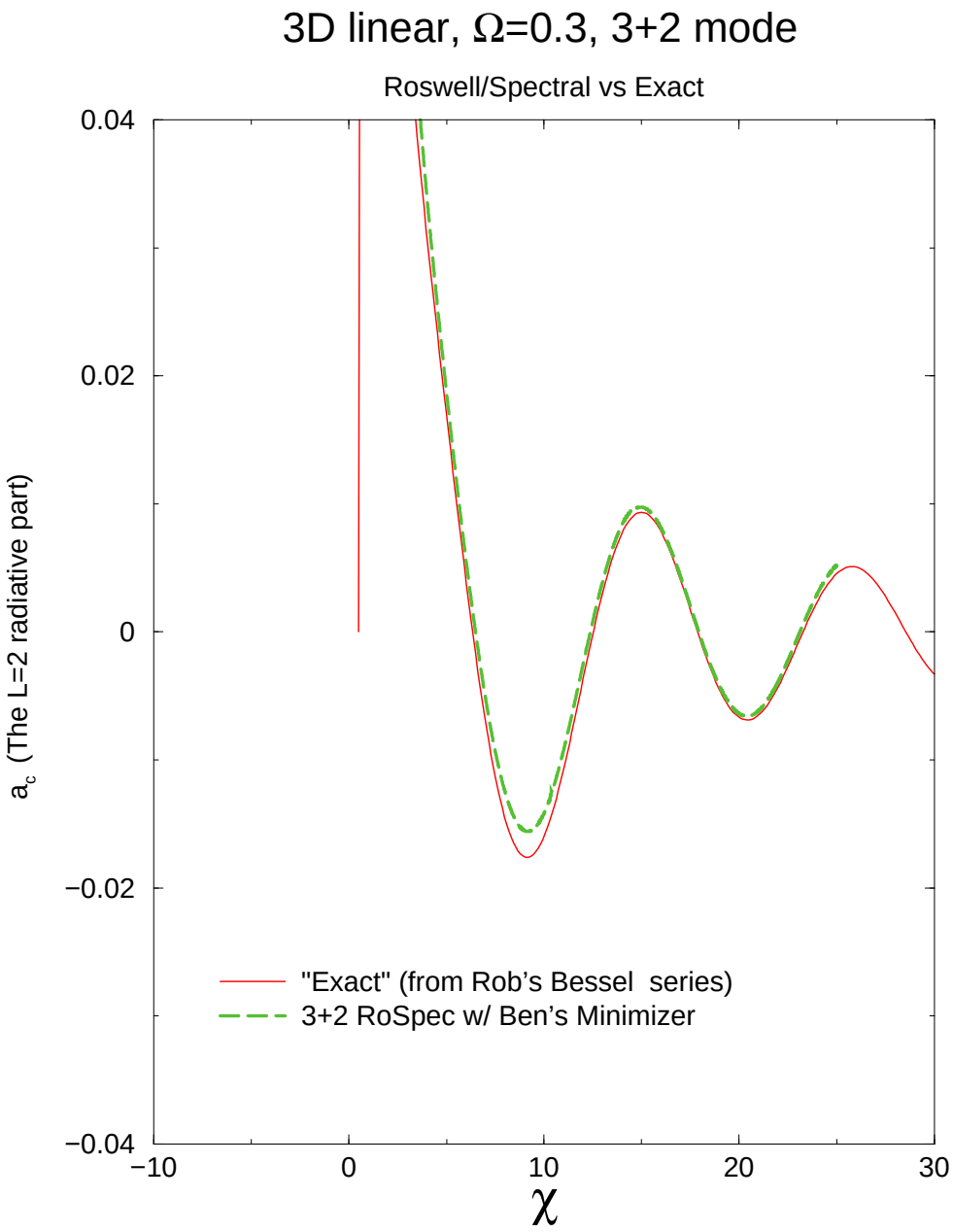
- Minimum amplitude
  - Balance of energy flow, not enough
  - Weak wave zone gives us notion of approximate multipoles; energy balance in each multipole not enough.
  - A degree of freedom remains; we can fix it by demanding minimum amplitude.

- Iteration

$$\mathcal{L}_{\Psi_n}\Psi_{n+1} = \text{sources}$$


χ=2
Φ=0
Θ=0.8
(a)
z Y
y X
x Z
(b)


First Nonlinear Results
λ=−40 α=2
0.04
0.02
0.00
−0.02
−0.04
amplitude
0
10
20
30
χ/a
standing
outgoing


3D linear, Ω=0.3, 3+2 mode
Roswell/Spectral vs Exact
0.04
0.02
0
−0.02
−0.04
−10
0
10
20
30
χ
a_c (The L=2 radiative part)
"Exact" (from Rob's Bessel series)
3+2 RoSpec w/ Ben's Minimizer