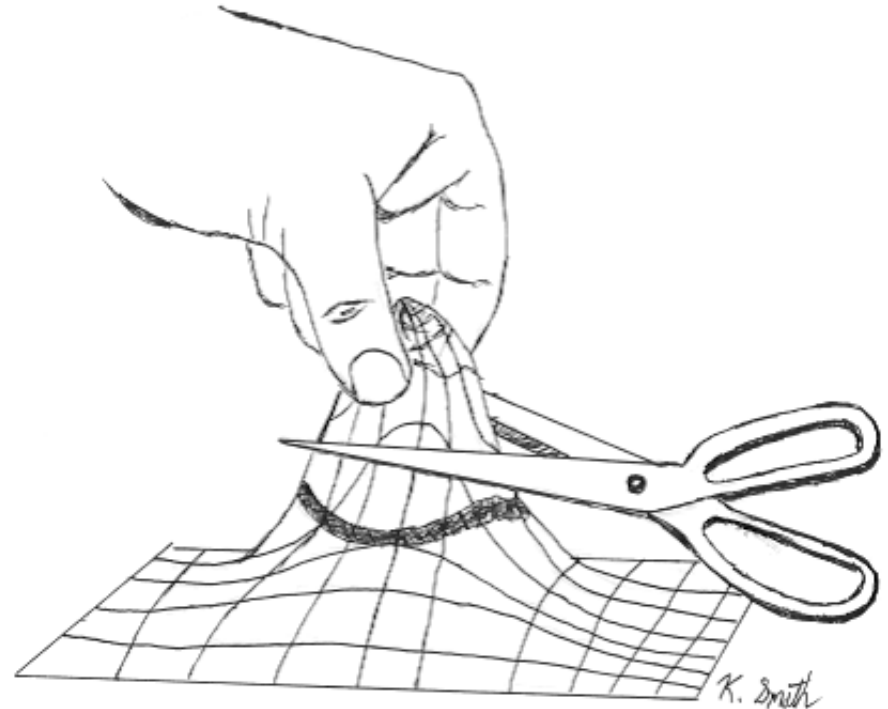




# NR @ Penn State

## Bouncing Black Holes Exercises in Dynamic Excision

*Kenneth L. Smith*  
*Penn State University*  
*12 December 2002*



# Thanks

Pablo Laguna  
Deirdre Shoemaker  
David Fiske  
Bernard Kelly  
Erik Schnetter  
Ulrich Sperhake

# Overview

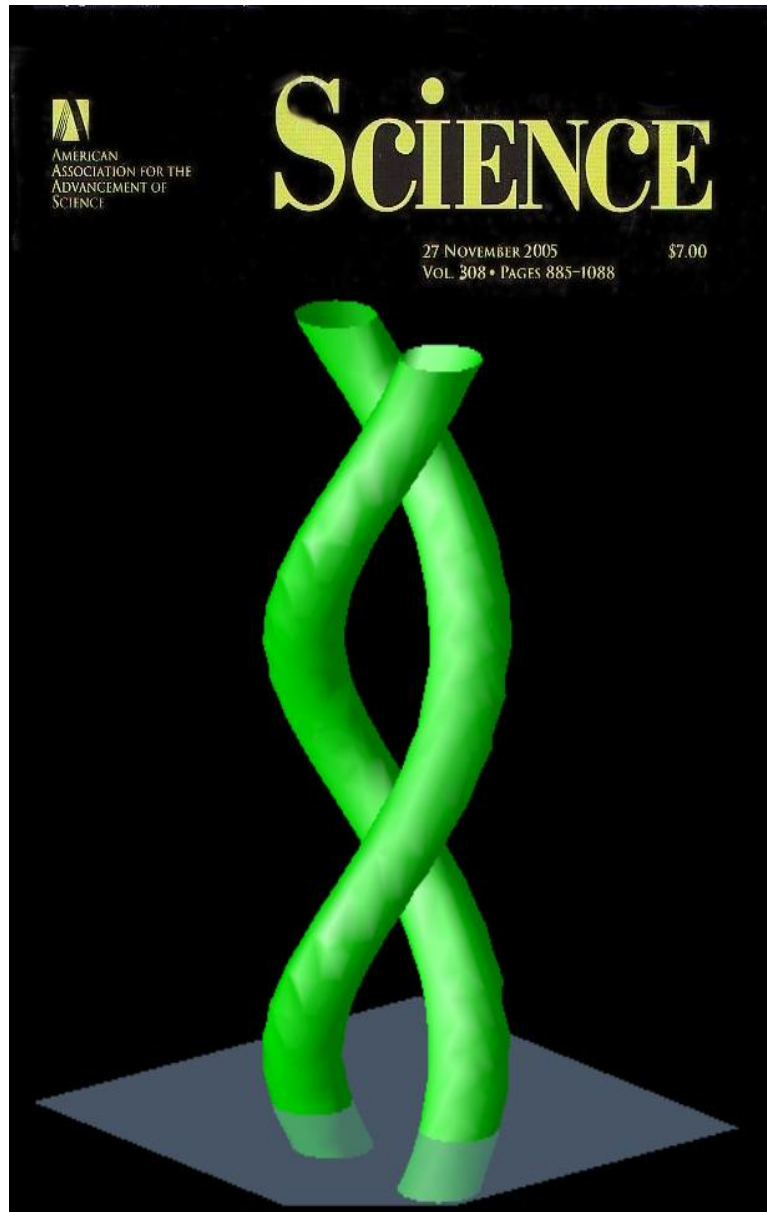
- *Motivation / Perspective*
- *Methods for handling singularities*
- *Excision in general*
- *Brief History*
- *Excision - the Maya way*
- *Test Problem*
- *Results*

# Some perspective



The cover of *Science*  
10 November 1995

# The future?



The cover of *Science*  
27 November 2005

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3D NR code:

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- Optimized, parallel 3D FD Cauchy code
- Your favorite formulation of equations
- Suitable, non-pathological gauge
- Consistent outer boundaries
- Astrophysically relevant initial data

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- Consistent outer boundaries
- Astrophysically relevant initial data

## ■ *The spacetime still contains physical singularities that must be treated in some way*

# Singularity-handling methods

- *Singularity-avoiding slicing conditions*
- *Puncture methods*
- *Inversion symmetry*
- *Strange matter*
- $\vdots$
- **Excision**

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- *If we must impose boundary conditions:*
  - Just insist that information goes into the hole

# A brief history

## ■ 1987: *Thornburg*

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## ■ *Success in 3D:*

- 1998: BBH Grand Challenge Head-on collision
- 2000: Pitt-PSU-Texas Grazing Collision (Agave)

# A brief history (cont'd)

## ■ 2001: Alcubierre & Brügmann

- No causal differencing
  - Centered differencing w/ upwind on advection ( $\beta^i \partial_i$ )
- Extrapolation B.C. (zeroth-order r.h.s. copy)
- Cubical excision

# A brief history (cont'd)

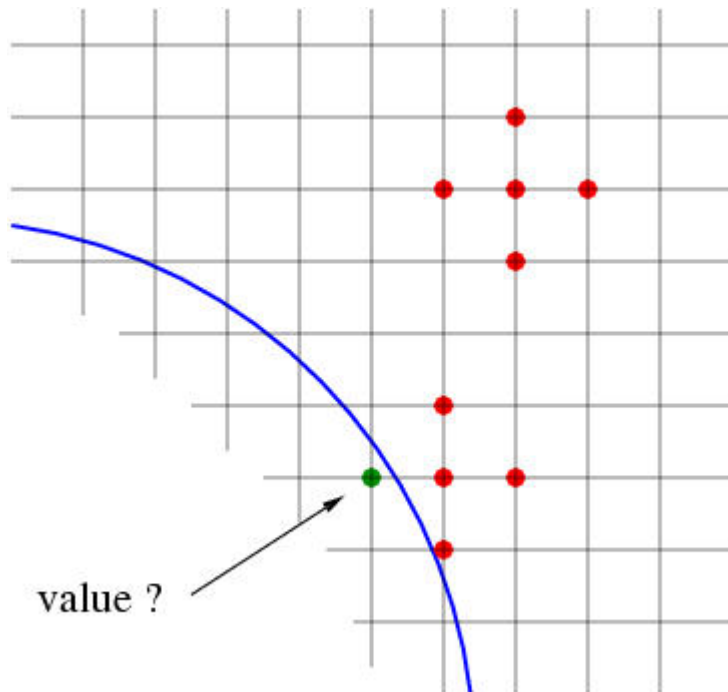
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## ■ 2001: Yo, Baumgarte, & Shapiro

- Extended to higher-order extrapolation
- Handled motion

# Excision in Maya



- Carry around an extra grid variable - excision mask
- Initially, label everything as being part of computational domain (**computational**)
- Select an excision shape, label points inside **excised**
- Relabel excised points that have **computational** points as neighbors **excised boundary**

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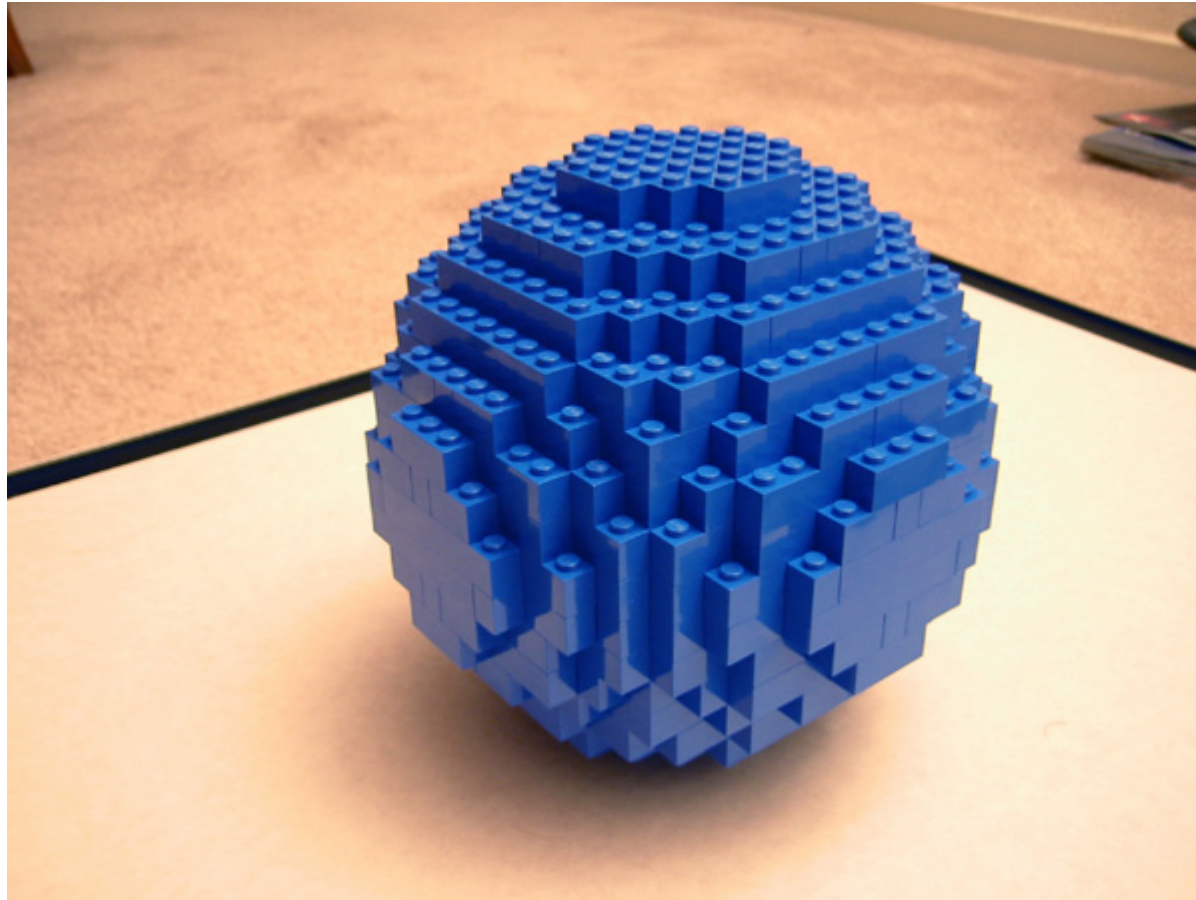
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  - Preserves symmetry of problem (e.g. octant, quadrant, bitant)

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- *Look to high school geometry!*

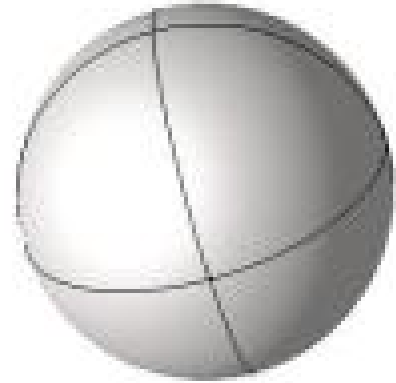
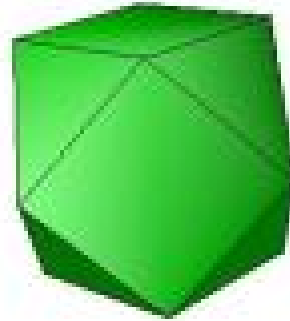
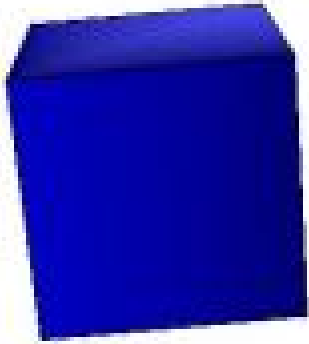
# The LEGO™ sphere

<http://www.google.com/search?q=lego+sphere>

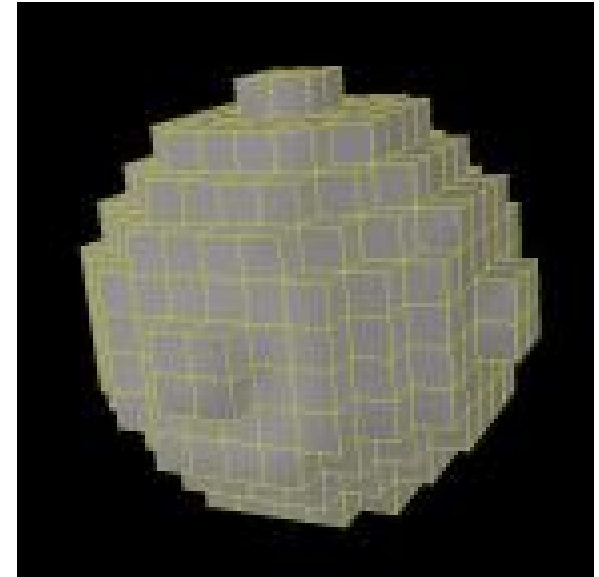
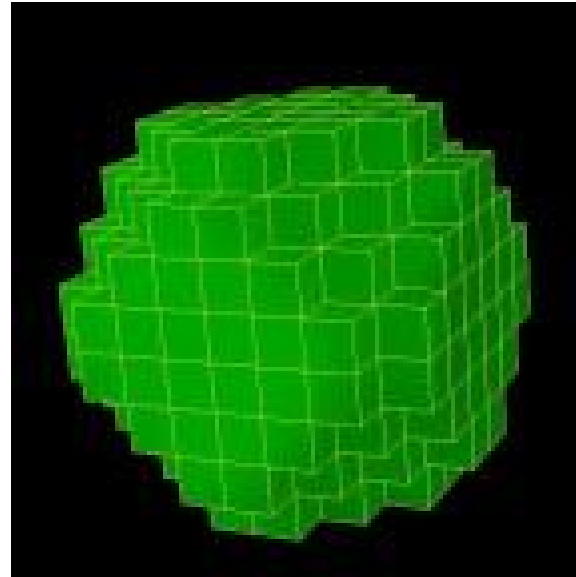
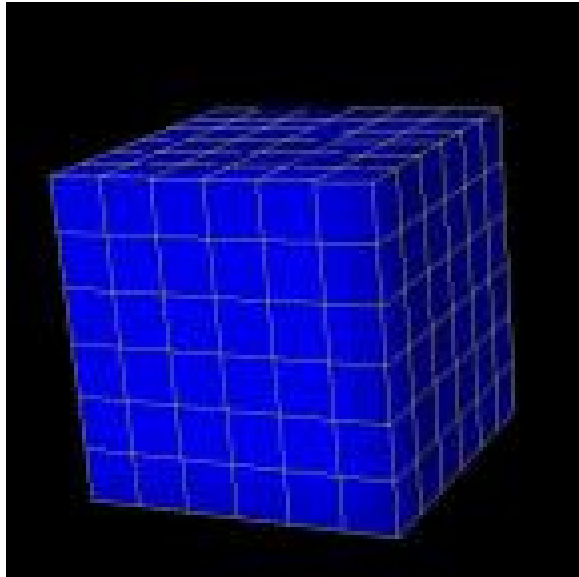
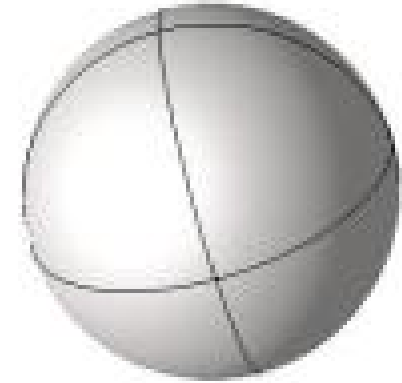
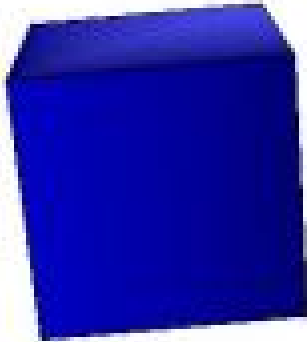


Credit: <http://www.brillig.com/lego/sphere>

# Excision shapes in Maya



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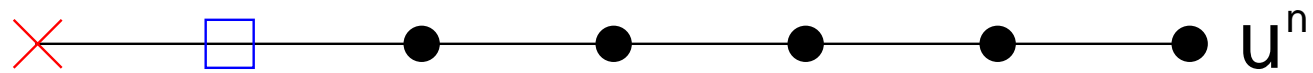
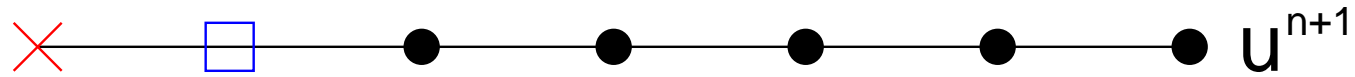
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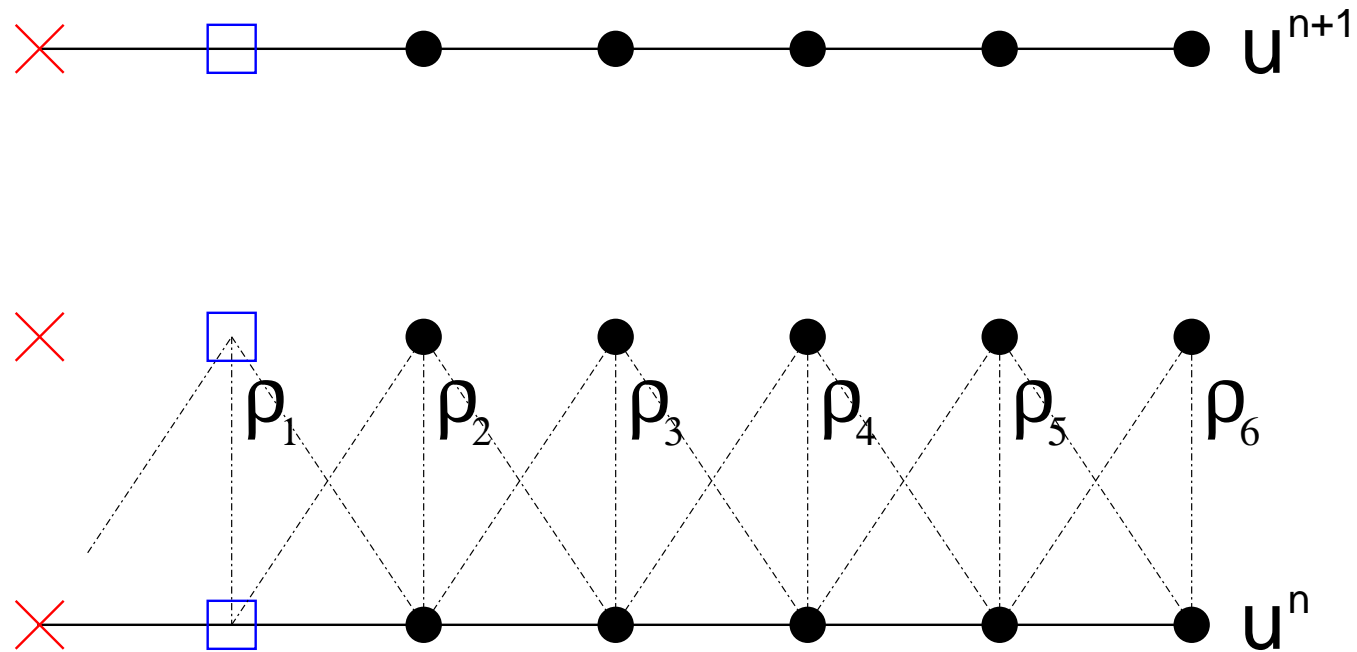
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$$f(i, j, k) = \sum_{n=1}^N a_n f(i_n, j_n, k_n)$$

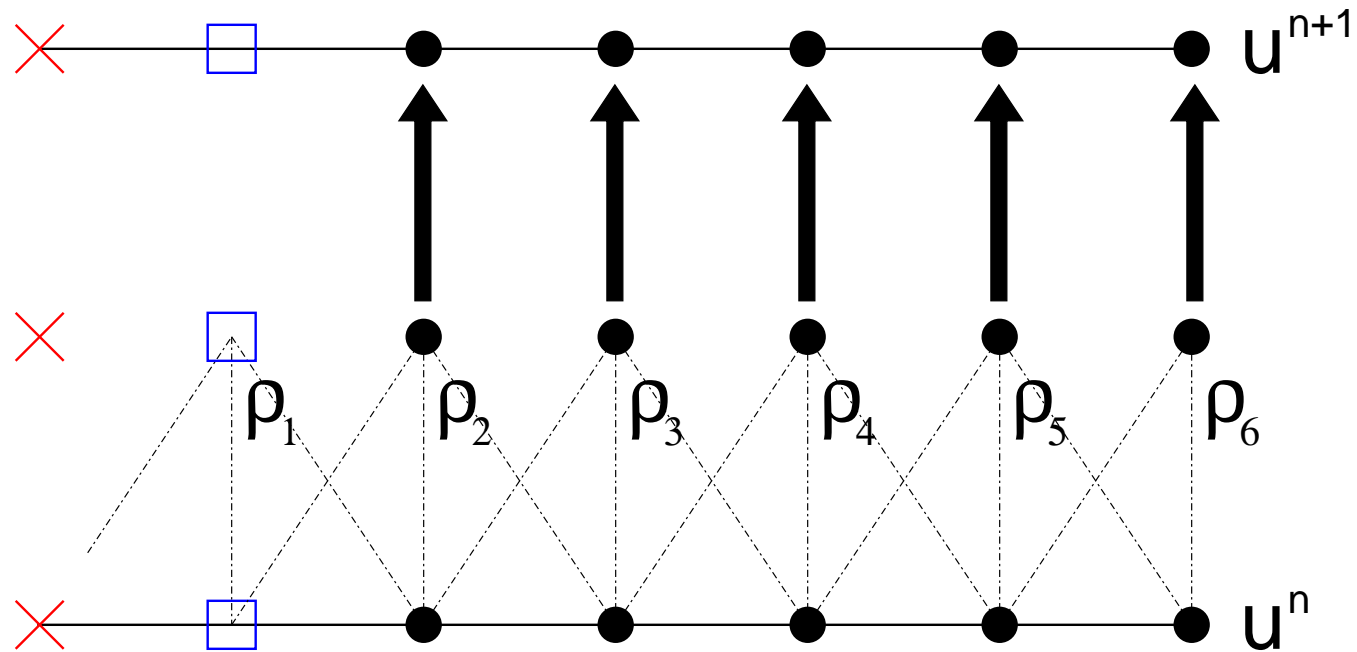
# An example of extrapolation



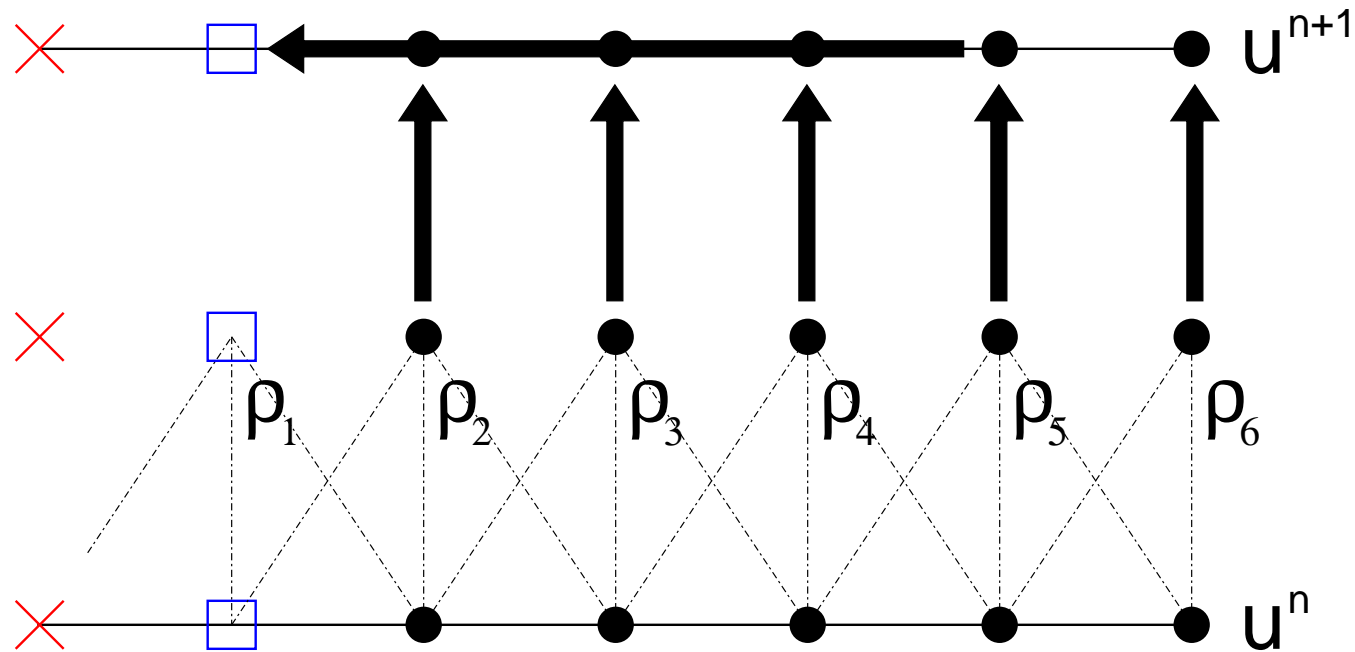
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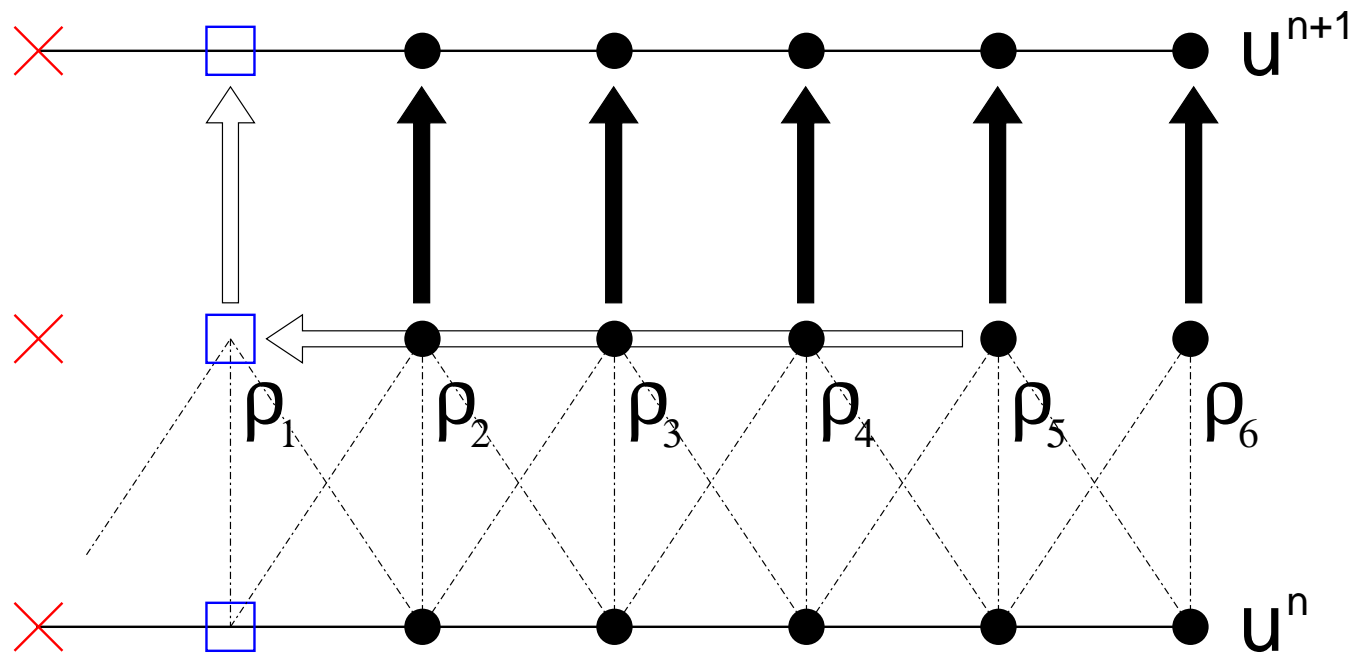
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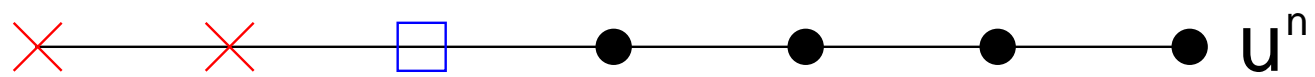
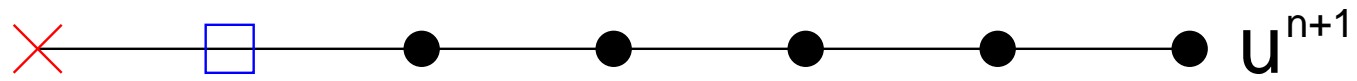
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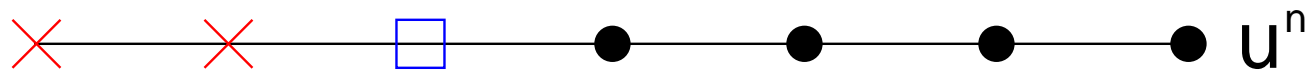
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$$g_{ij} = \delta_{ij} + \frac{2M}{\bar{r}} \frac{\bar{x}_i \bar{x}_j}{\bar{r}}$$
$$K_{ij} = \frac{2M}{\bar{r}^2} \left(1 + \frac{2M}{\bar{r}}\right)^{-1/2} \left[ \delta_{ij} - \frac{2\bar{r} + M}{\bar{r}} \frac{\bar{x}_i \bar{x}_j}{\bar{r}} \right]$$
$$\alpha = \left(1 + \frac{2M}{\bar{r}}\right)^{-1/2}$$
$$\beta^i = \frac{2M}{\bar{r}} \left(1 + \frac{2M}{\bar{r}}\right)^{-1} \frac{\bar{x}^i}{\bar{r}} - \dot{x}_{BH}^i$$

# Simulations

- *BSSN evolution*
- *Analytic lapse and shift*  
( $\alpha(t)$  &  $\beta(t)$ )
- *Excision radius:  $1.5M$*
- *Extrapolation:*  
*3rd order soln*
- *Outer boundary: Analytic*
- $dx^i = 0.20M$
- $dt = 0.05M$
- *Bitant symmetry*
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# Results

- [http://www.phys.psu.edu/~ksmith/talks/2002/nrlunch/stationary\\_tracek.mpg](http://www.phys.psu.edu/~ksmith/talks/2002/nrlunch/stationary_tracek.mpg)
- [http://www.phys.psu.edu/~ksmith/talks/2002/nrlunch/stationary\\_nham.mpg](http://www.phys.psu.edu/~ksmith/talks/2002/nrlunch/stationary_nham.mpg)
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