# Finding Event Horizons in 3D Numerical Spacetimes

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# Outline

- Properties of Event Horizons
- Methods of Finding Event Horizons
- Numerical Implementation
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# Properties of Event Horizons I

The event horizon is defined as the boundary of the causal past of future null infinity i.e. it is the (2+1) surface in (3+1) space inside of which no null geodesics have a future at future null infinity, and outside of which at least some do.

In principle it is necessary to know the full space-time in order to find the event horizon (in contrast the apparent horizon is a local concept).

The event horizon is generated by null geodesics. This implies that the event horizon is (almost everywhere) a null surface. These points are referred to as caustics.

Except at these points the event horizon is normally smooth and differentiable.

# Properties of Event Horizons II

Case study: Radially outgoing null geodesics in Schwarzschild

$$\frac{dr}{dt} = 1 - \frac{2M}{r}$$

Look at outgoing null geodesics near the horizon: $r = 2M + \epsilon$

$$\frac{d\epsilon}{dt} = 1 - \frac{2M}{2M + \epsilon} = 1 - \frac{1}{1 + \epsilon/(2M)} \approx \frac{\epsilon}{2M}$$

So near the horizon

$$r(t) = 2M + \epsilon_0 \exp\left(\frac{t}{2M}\right) = 2M + \epsilon_0 \exp\left(\frac{t}{\lambda}\right)$$

Similar results are obtained for example for Kerr-Schild and isotropic coordinates ($\lambda = 4M$).

# Methods of Finding Event Horizons I

Integrating the geodesic equations for individual photons:

$$\frac{d^2x^\alpha}{d\lambda^2} = -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{d\lambda}\frac{dx^\gamma}{d\lambda}.$$

1. Shooting photons in all directions from all points to find out if any of the photons escape to infinity (Hughes et. al., 1994).
   Extremely time consuming.

2. Integrate photons backwards in time.
   Problem with tangential drift.

Common numerical issue: Both the metric and its first derivatives has to be interpolated to the current location of the geodesics.

# Methods of Finding Event Horizons II

Define the horizon as a surface in 4-space by a level set function

$$f(t, x^i) = 0,$$

with the normal $n_\mu = \partial_\mu f(t, x^i)$.The requirement that the surface is null, amounts to $n_\mu n^\mu = 0$ or

$$g^{\mu\nu} n_\mu n_\nu = g^{\mu\nu} \partial_\mu f \partial_\nu f = g^{tt} (\partial_t f)^2 + 2 g^{ti} \partial_t f \partial_i f + g^{ij} \partial_i f \partial_j f = 0.$$

This is a quadratic equation in $\partial_t f(t, x^i)$. Solving it gives an evolution equation for $f(t, x^i)$

$$\partial_t f = \frac{-g^{ti} \partial_i f + \sqrt{(g^{ti} \partial_i f)^2 - g^{tt} g^{ij} \partial_i f \partial_j f}}{g^{tt}}.$$

# Methods of Finding Event Horizons III

The algorithm for finding the event horizon is:

1. Evolve numerically until the spacetime is almost stationary outputting enough data to be able to reconstruct the 4-metric at each time slice.
2. Choose two surfaces: One inside and one outside of the event horizon, trapping the event horizon between them.
3. Evolve these surfaces backwards in time.
4. When the distance between these two becomes small enough, the event horizon has been located and can be tracked until the initial data slice is reached.

# Numerical Implementation I

The event horizon finder is implemented as a thorn in the Cactus Computational Toolkit.

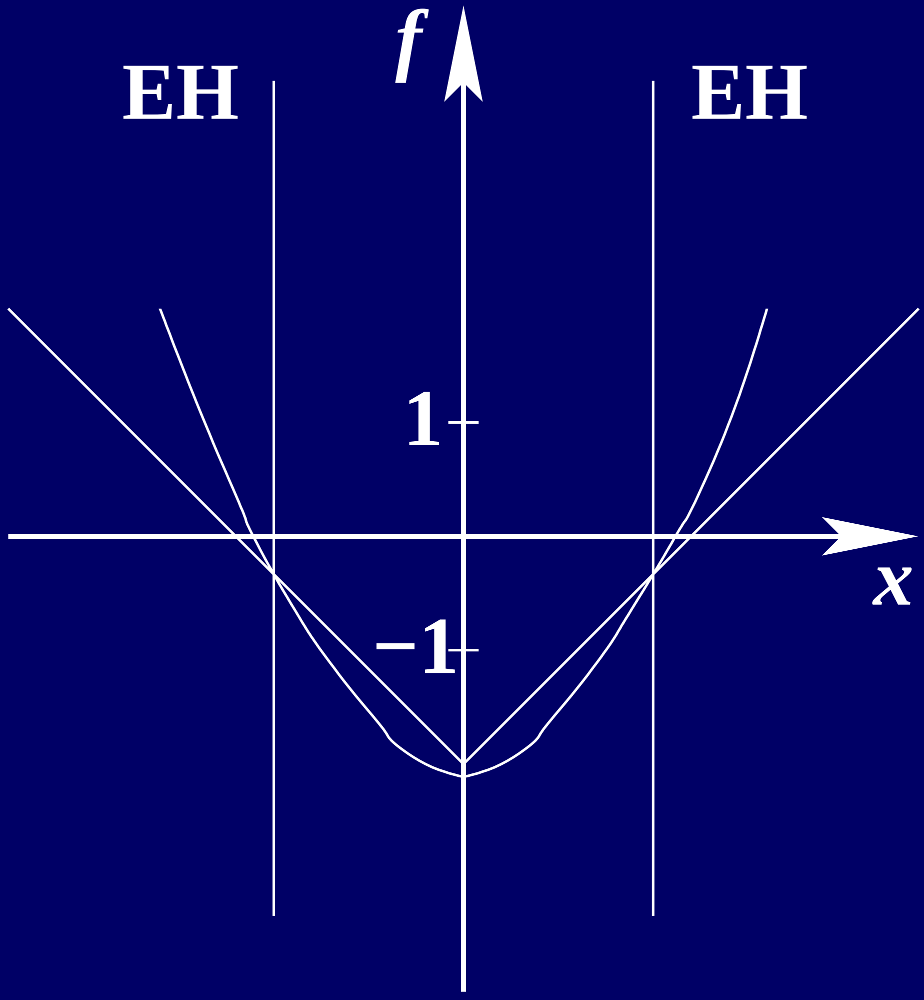
The equation

$$\partial_t f = \frac{-g^{ti}\partial_i f + \sqrt{(g^{ti}\partial_i f)^2 - g^{tt}g^{ij}\partial_i f \partial_j f}}{g^{tt}}$$

is evolved in time using the Method of Lines (MoL) with either a second order Runge-Kutta or Iterative Crank Nicholson scheme.

Numerical problems arise if this scheme is implemented naively with standard centered second order finite differencing.

# Numerical Implementation II




Steepening of gradients is bad for numerics. Re-initialization of $f$ to a distance function is needed.

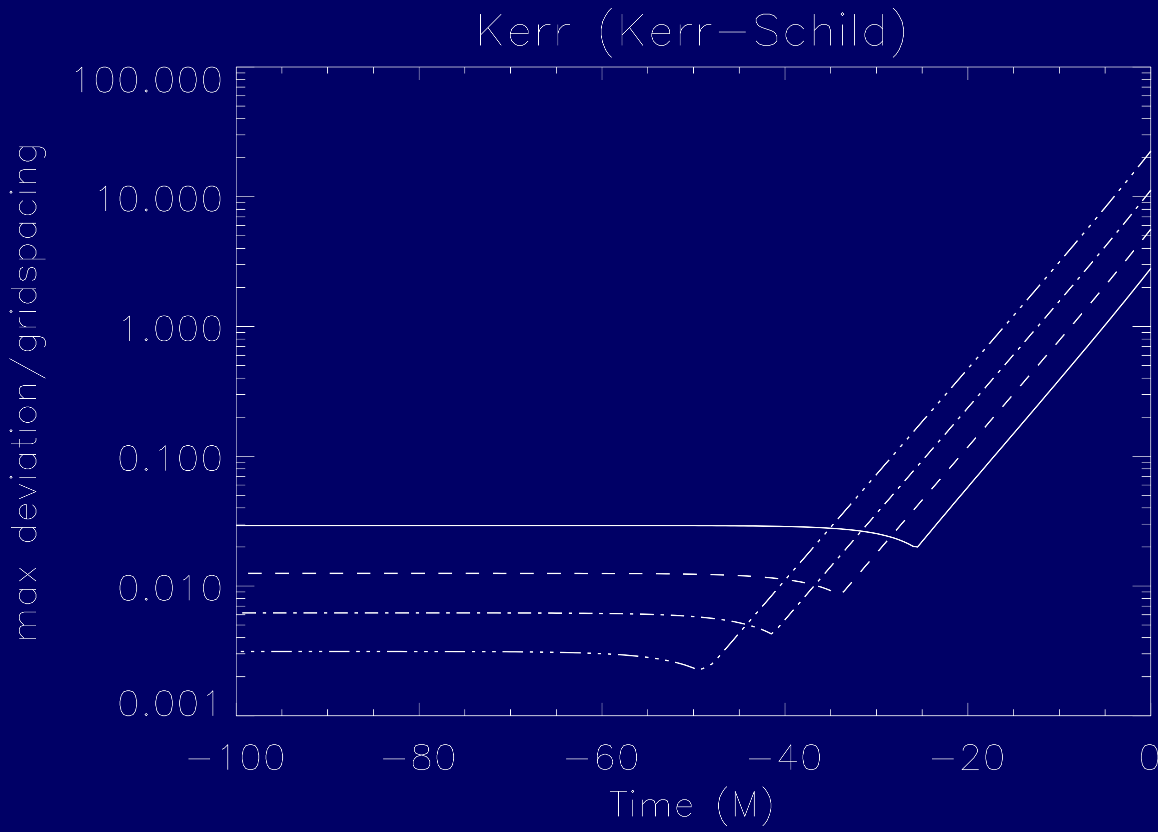
$$\frac{df}{d\lambda} = -\frac{f}{\sqrt{f^2+1}}\left(|\nabla f| - 1\right)$$

is evolved until a steady state is reached.

Close to topology changes the re-initialization can move the surface slightly.

# Tests I

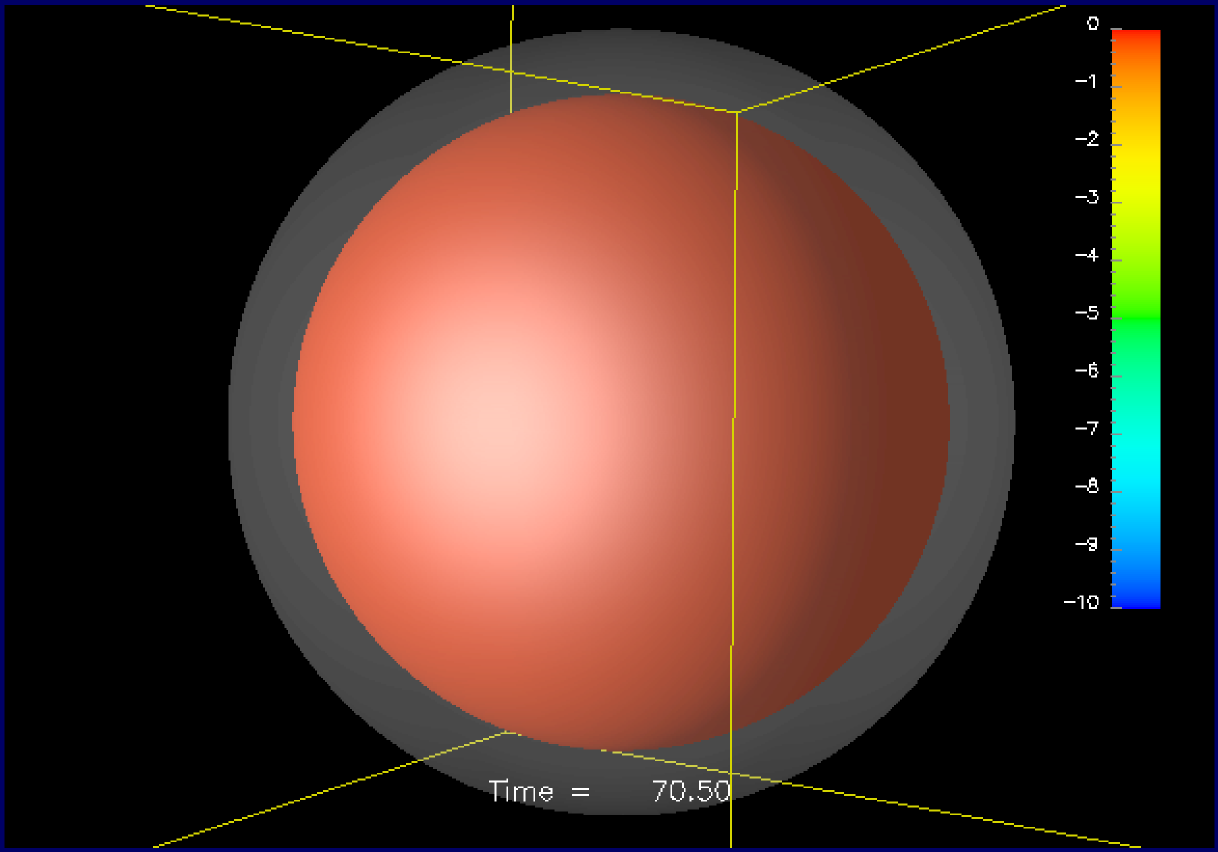
Kerr ($M = 1$, $a/M = 0.8$) in Kerr-Schild coordinates ($\Delta = 0.2,\ 0.1,\ 0.05,\ 0.025$).




$$\lambda_{num} = 5.3M,\ \|d\|_\infty/\Delta = 0.029, 0.013, 0.0062, 0.0031$$

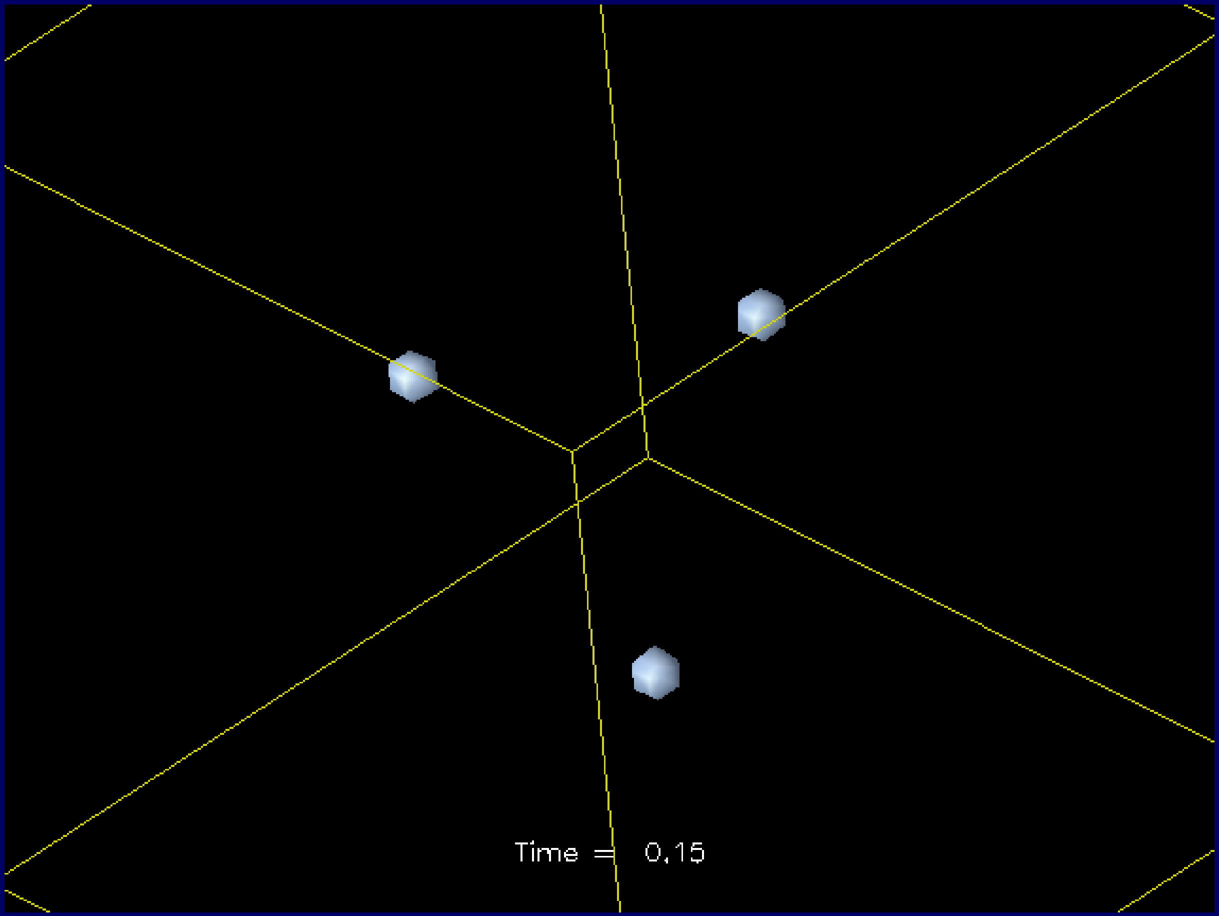
# Tests II

Head on collision of two black holes (Misner $\mu = 2.2$ initial data, $L = 4.46M$).



# Tests III

Time symmetric 3 black hole collision (Brill-Lindquist initial data).



# Horizon Analysis and Future Work

It's not enough just to find the event horizon! We also want to do some physics with it. That means we need analysis tools like:

- Horizon area (Coded. Being tested.)
- Horizon circumference (Coded. Being tested.)
- Gaussian Curvature (Not yet coded.)
- Horizon generators (Not yet coded.)
- Expansion of the congruence of horizon generators (Not yet coded.)
- Examine the horizon properties for all spacetimes that we can evolve long enough.