## Questions for Observational Cosmology

- What is the age of the universe implied for the observed expansion rate? How does this match observations of the oldest stars in the universe?
- What are the values of the cosmological parameters $H_0$, $\Omega_0$ (or $q_0$), and $\Lambda$?
- What is the large scale distribution of matter in the universe?
- What is the dark matter? How good is the evidence for it?
- What is the evidence for dark energy? How good is it?
- What is the nature of the cosmic background radiation? What sort of constraints does it place on some of the above questions?
- Can we come up with a consistent theory for the formation of the structures that we see in the universe? How do sheets, walls, voids, clusters, and galaxies form?
- Lots more, but these are the ones we'll try to address this semester!

## The Robertson-Walker Metric

General relativity is a geometric theory describing the Curvature of space time. We want to describe the distance between two events (an event happens at a certain place AND time) in a 4-dimensional spacetime.

In 3-dimensional space, we can describe the separation between two points as:

$$ds^2 = du^2 = dx^2 + dy^2 + dz^2$$

The distance between two points is found by integrating along the path. In spacetime the interval between these two points is:

$$ds^2 = c^2 dt^2 - du^2$$

And we have to integrate along time and space to find the distance.

## The Robertson-Walker Metric

In the case of an expanding universe, the space distance between two points is:

$$ds^2 = c^2 dt^2 - R^2 du^2$$

In spherical coordinates in Cartesian geometry:

$$du^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

But space-time isn't necessarily flat, it may have curvature (remember k?) so:

$$du^2 = \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2$$

## The Robertson-Walker Metric

Putting this all together, we have:

$$\begin{aligned} ds^2 &= c^2 dt^2 - R^2 du^2 \\ &= c^2 dt^2 - R^2 \left\{ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right\} \end{aligned}$$

r is the co-moving coordinate, R(t) is the scale factor of the expansion and $d_p$ = R(t) x r is the "proper distance", and k is the curvature. These can be observationally determined.

This is known as the Robertson-Walker metric discovered independently in the mid-1930's. Note that it is physics free and does not depend on gravitational theory which enters through the factors R(t) and k.

## Distance measures in an Expanding Universe

We define the proper distance as $d_p = \int R du$. For light, ds = 0, and we can use the RW metric to define:

$$c^2 dt^2 = R^2 du^2 = R^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \right\}$$

To simplify things, let's put ourselves at the origin, then The light's path is purely radial, and $d\theta$ and $d\phi$ =0.

$$c^2 dt^2 = R^2 \left\{ \frac{dr^2}{1-kr^2} \right\}$$

Taking the square root of both sides and integrating:

$$\int_{t_0}^{t_1} \frac{c}{R} dt = \int_u^0 \frac{dr}{(1-kr^2)^{1/2}}$$

## Distance measures in an Expanding Universe

$$\int_{t_0}^{t_1} \frac{c}{R} dt = \int_u^0 \frac{dr}{(1-kr^2)^{1/2}}$$

In general this is non-analytic. In a $\Lambda$=0 universe:

$$d_p = \frac{c}{H_0 q_0^2 (1+z)} \left\{ q_0 z + (q_0 - 1) \left[ (2q_0 z + 1)^{1/2} - 1 \right] \right\}$$

For non-zero $\Lambda$ universe:

$$d_p = |\Omega_k|^{-\frac{1}{2}} \sinh \left\{ |\Omega_k|^{\frac{1}{2}} \int_0^z \left\{ (1+z)^2 (1+\Omega_M z) - \Omega_\Lambda z(2+z) \right\}^{\frac{1}{2}} dz \right\}$$

Assuming $\Omega_k$<0, if $\Omega_k$ >0 then the sinh becomes a sin and if $\Omega_k$=0 then the sinh and the $\Omega_k$ drop out and all that's left is the integral. This definitely non-analytic!

## Distance measures in an Expanding Universe

Let's assume that a radiant flux F is observed for a source of light with known bolometric luminosity L. We define the luminosity distance $d_L$ of the object by:

$d_L^2 = L/4\pi F$ or $F=L/ 4\pi d_L^2$

in an expanding universe, two effects reduce the measured flux. The cosmological redshift reduces the energy of the photon (=$h\nu$) by 1+z. Cosmological time dilation affects the time interval between photons, so their arrival rate is reduced by 1+z.

We know that the area of the sphere that the photons cover is $4\pi d_p^2$ so $F = L/(1+z)^2 4\pi d_p^2$ or

$d_L = (1+z) d_p$

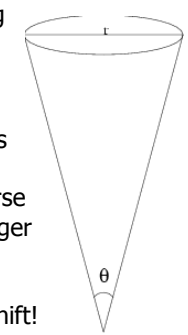
## Distance measures in an Expanding Universe

Similarly we can define the angular diameter distance as the distance needed to reproduce the Euclidean result:

$$\theta = \frac{s}{d_A}$$

Where s is the size of the galaxy and $\theta$ is its observed angular size. We can also determine $d_A$ by integrating the RW metric. We find that:

$$d_A = d_p (1+z)^{-1} = d_L (1+z)^{-2}$$

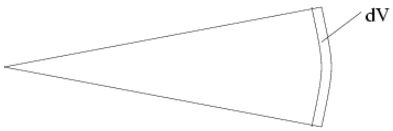



This also means that the angle subtended by a galaxy is proportional to s/R (the fractional size of the galaxy relative to the universe. At early times when the universe was small, our "standard yardstick" extended over a larger fraction of the universe than it does today so it appears larger viewed today. So observed angular sizes of "standard yardsticks" can actually increase at high redshift!

Also note that since surface brightness is $f/\theta^2$ it decreases by $(1+z)^{-4}$. High-redshift galaxies are really hard to observe!!

## Volume element in an Expanding Universe

We also want to determine the differential volume element (dV) as a function of distance (or redshift).




In Euclidean space: $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$

In general we need to account for spacetime and the expansion of the universe. This is (again!) non-analytic. For $\Lambda$=0:

$$dV = r_A^2 \frac{c}{H_0 \left(2q_0 z + 1\right)^{1/2} (1+z)^2} \sin\theta \, dz \, d\theta \, d\phi$$

## Lookback time and the age of the universe:

We saw last time that for a $\Lambda$=0 universe the age of the universe, $t_0$, is somewhere between 2/3 ($1/H_0$) and $1/H_0$. We can also define the lookback time as how far back we are looking when we observe an object at a redshfit z, $t_L = t_0 - t(z)$ where t(z) is the age of the universe at t(z). Lookback time and age can be determined by integrating the Friedmann equation. These equations are (again!) in general non-analytic, but note that to within a few % for non-zero $\Lambda$ universe:

$$t_0 \approx \frac{2}{3} H_0^{-1} \frac{\sinh^{-1} \sqrt{|1-\Omega_a|/\Omega_a}}{\sqrt{|1-\Omega_a|}}$$

$$\Omega_a = \Omega_M - 0.3\Omega_\Lambda + 0.3$$

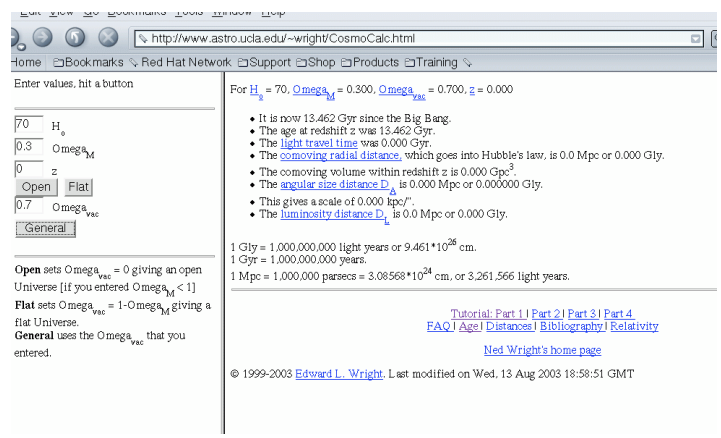
## What happens in various cosmological models?

In order to get a feel for the effects of cosmology on these parameters, plotting the values for various models is extremely helpful!

| Model | $\Omega_m$ | $\Omega_\Lambda$ |
|---|---|---|
| Einstein-de Sitter | 1 | 0 |
| Low density | 0.05 | 0 |
| High $\Lambda$ | 0.2 | 0.8 |

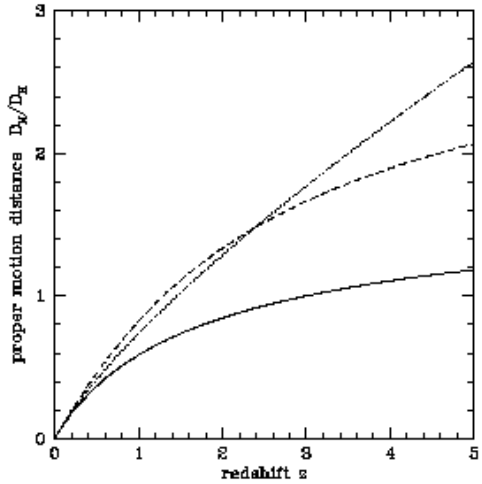
## How do we do these calculations?

Write your own code to do the numerical integrations, look them up, OR use Ned Wright's Cosmology Caluclator!



## Proper distance

$D_H = c/H_0$




$\Omega_M,\Omega_\Lambda$=0.05,0

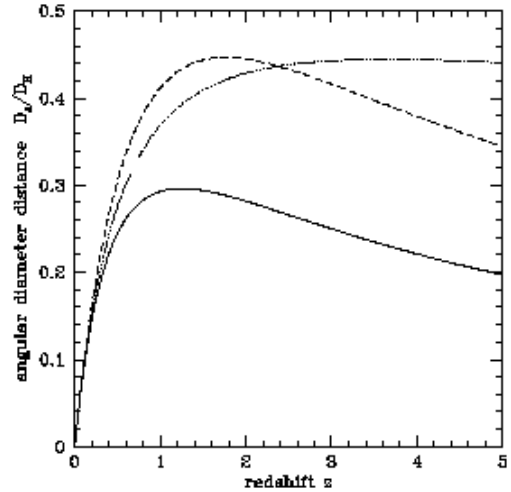
$\Omega_M,\Omega_\Lambda$=0.2,0.8

$\Omega_M,\Omega_\Lambda$=1,0

Plots from Hogg (2000)

## Angular diameter distance

$D_H = c/H_0$

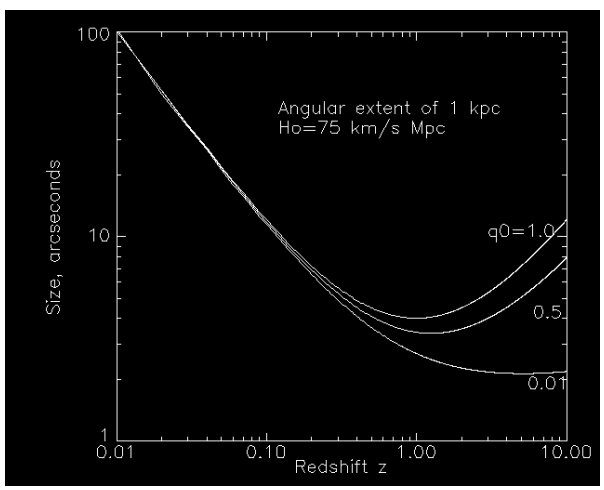



$\Omega_M,\Omega_\Lambda$=0.05,0

$\Omega_M,\Omega_\Lambda$=0.2,0.8

$\Omega_M,\Omega_\Lambda$=1,0

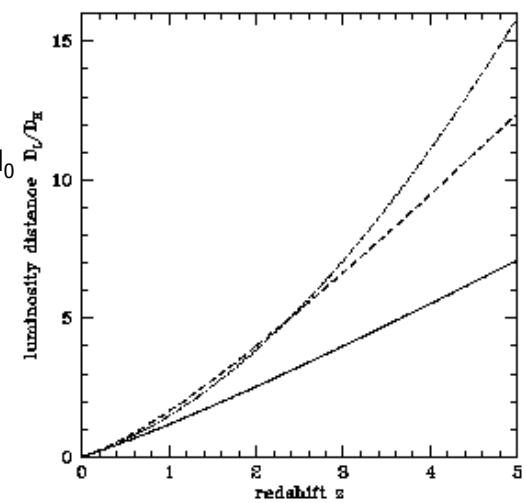
## Angular-size redshift test




In practice, it is hard to find a standard yardstick. Why??

## Luminosity distance

$D_H = c/H_0$



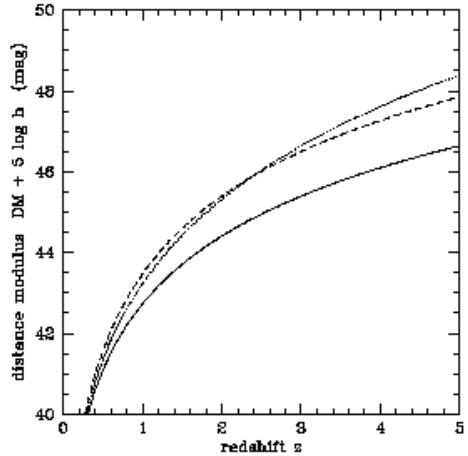

$\Omega_M,\Omega_\Lambda$=0.05,0

$\Omega_M,\Omega_\Lambda$=0.2,0.8

$\Omega_M,\Omega_\Lambda$=1,0

## Distance modulus - redshift

DM=m-M



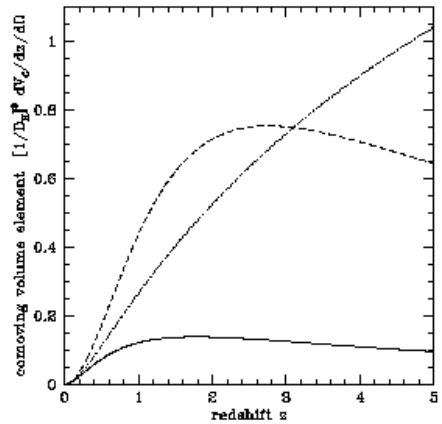

$\Omega_M,\Omega_\Lambda$=0.05,0

$\Omega_M,\Omega_\Lambda$=0.2,0.8

$\Omega_M,\Omega_\Lambda$=1,0

Note this can be a relative test, using the "redshift-magnitude" relation also known as the "Hubble Diagram"

## Comoving volume element




$\Omega_M,\Omega_\Lambda$=0.05,0

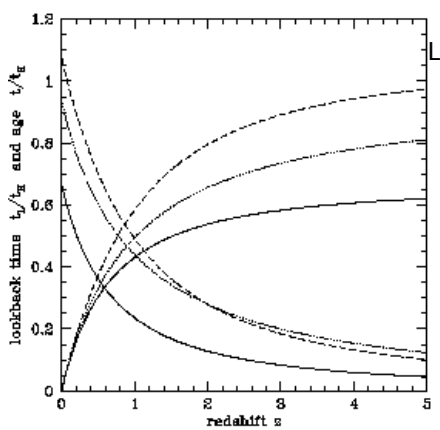
$\Omega_M,\Omega_\Lambda$=0.2,0.8

$\Omega_M,\Omega_\Lambda$=1,0

So, theoretically the number counts of galaxies versus redshift (actually apparent magnitude, traditionally) should be a great cosmological test. Why doesn't it work??

## Lookback time and age in units of $t_H=1/H_0$

Age




Lookback time

$\Omega_M,\Omega_\Lambda$=0.2,0.8

$\Omega_M,\Omega_\Lambda$=0.05,0

$\Omega_M,\Omega_\Lambda$=1,0

For $\Omega_M,\Omega_\Lambda$=0.05,0 and $H_0$=70: age=12.5 Gyr
For $\Omega_M,\Omega_\Lambda$=1,0 and $H_0$=70: age=9.3 Gyr
For $\Omega_M,\Omega_\Lambda$=0.2,0.8 and $H_0$=70: age=15.02 Gyr
<u>** For $\Omega_M,\Omega_\Lambda$=0.3,0.7 and $H_0$=70: age=13.46 Gyr</u>
this is the "benchmark" model in S+G

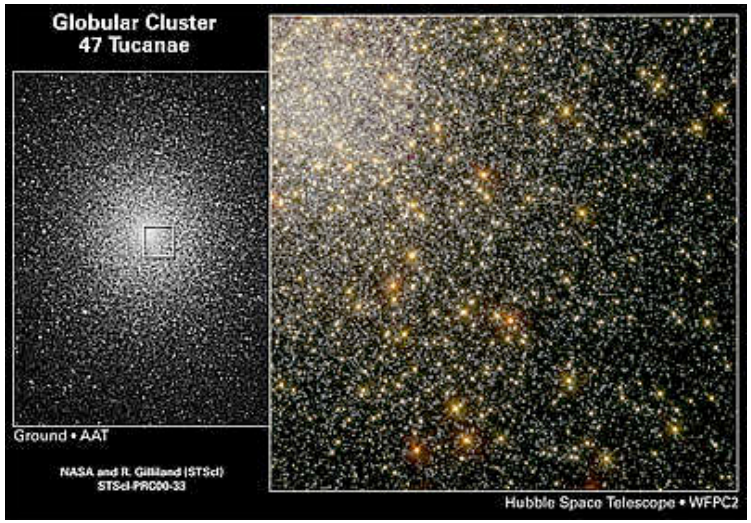
What if $H_0$=50?
For $\Omega_M,\Omega_\Lambda$=0.05,0: age=17.5 Gyr
For $\Omega_M,\Omega_\Lambda$=1,0: age=13.04 Gyr

# Age of the universe

- One of the fundamental questions in cosmology is how old is the universe?
  - Ages of the oldest stars – globular clusters, white dwarfs, nucleocosmochronology
  - Age of galaxies at high redshift
  - Expansion age of the universe derived from cosmological parameters
  - Do they agree or not?

# Globular clusters

- They are among the oldest objects in the galaxy, provide a lower limit on the age of the universe
  - Why is it a lower limit?
  - There are a fair number of uncertainties in these estimates, including errors in measuring the distances to the GCs and uncertainties in the isochrones used to derive ages (i.e. stellar evolution models)
  - Inputs to stellar evolution models include – oxygen abundance [O/Fe], treatment of convection, helium abundance, reaction rates of $^{14}N + p \rightarrow {}^{15}0 + \gamma$, helium diffusion, conversions from theoretical temperatures and luminosities to observed colors and magnitudes, and opacities




47 Tucanae

# Ages of globular clusters

- We measure the age of a globular cluster by measuring the magnitude of the main sequence turnoff
- Compare this to stellar evolutionary models which estimate the surface temperature and luminosity of a stars as a function of time

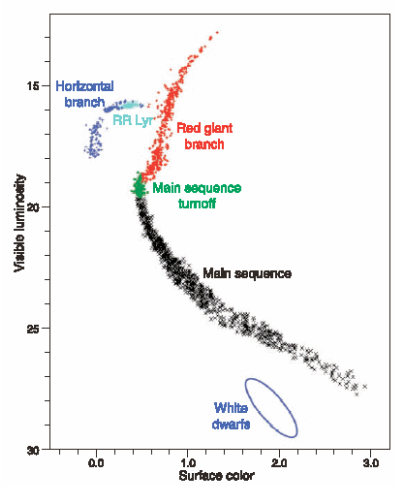
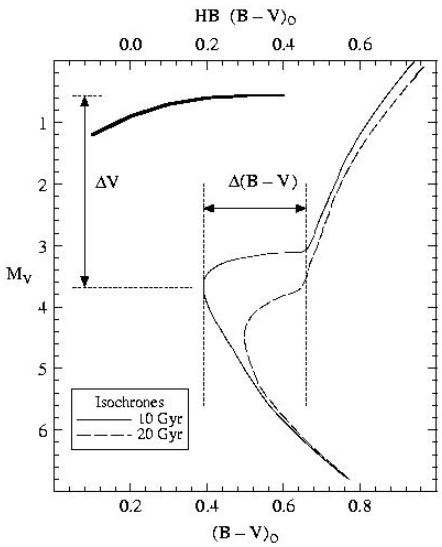
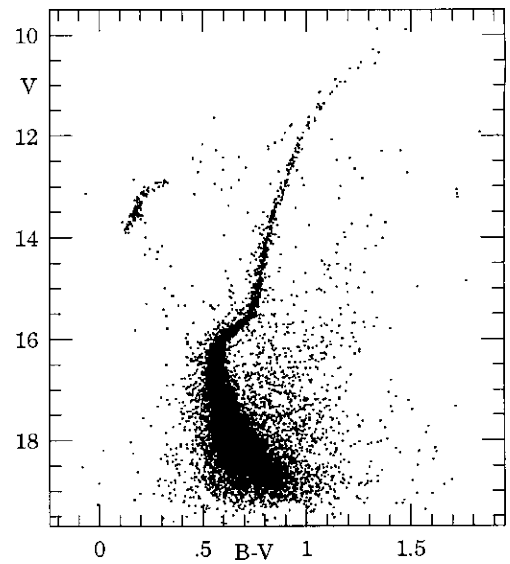



Fig. 2. A schematic color-magnitude diagram for a typical globular cluster (33) showing the location of the principal stellar evolutionary sequences. This diagram plots the visible luminosity of the star (measured in magnitudes) as a function of the surface color of the star (measured in B-V magnitude). Hydrogen-burning stars on the main sequence eventually exhaust the hydrogen in their cores (main sequence turnoff). After this, stars generate energy through hydrogen fusion in a shell surrounding an inert hydrogen core. The surface of the star expands and cools (red giant branch). Eventually the helium core becomes so hot and dense that the star ignites helium fusion in its core (horizontal branch). A subclass is unstable to radial pulsations (RR Lyrae). When a typical globular cluster star exhausts its supply of helium, and fusion processes cease, it evolves to become a white dwarf.

Typical globular cluster color magnitude diagram (CMD)




Measuring the age of a GC.
Note ΔV is reddening insensitive.




NGC 6397, Kaluzny (1997)

# Recent Results for globular clusters

- Gratton et al (2003) find
  - NGC 6397 & NGC 6752 (halo globular clusters) are 13.9 +/- 1.1 and 13.8 +/- 1.1 Gyr old (without helium diffusion)
  - 47 Tuc (a disk globular cluster) is 11.3 +/- 1.1 Gyr old
  - With diffusion, find ages of 13.4 +/- 0.8 (random errors) +/- 0.6 (systematic errors) Gyr for the oldest clusters
  - What is the full allowed range with all the possible inputs?

**Table 1.** Estimates of $M_v$(RR).

| Method | Objects | [Fe/H] | $M_v$(RR) | $M_v$(RR) at [Fe/H] = −1.90 |
|---|---|---|---|---|
| Theoretical HB models | Metal-poor blue HB clusters | −1.90 | 0.35 ± 0.10 | 0.35 ± 0.10 (*19*) |
| Main sequence fitting | M13, M68, M92, N6752 | −1.83 | 0.35 ± 0.10 | 0.36 ± 0.10 (*16*) |
| White dwarf fitting | N6397 | −1.42 | 0.52 ± 0.15 | 0.41 ± 0.15 (*15*) |
| LMC RR Lyr | RR Lyr in the LMC | −1.90 | 0.44 ± 0.10 | 0.44 ± 0.10 (*34*) |
| Trigonometric parallax | RR Lyrae | −1.39 | 0.61 ± 0.11 | 0.49 ± 0.15* (*21*) |
| Trigonometric parallax | Field halo HB stars | −1.51 | 0.60 ± 0.12 | 0.51 ± 0.16* (*17*) |
| Dynamical | M2, M13, M22, M92 | −1.74 | 0.59 ± 0.15 | 0.55 ± 0.15 (*16, 35*) |
| Statistical parallax | Field RR Lyr | −1.60 | 0.77 ± 0.13 | 0.71 ± 0.16* (*36*) |

*Includes additional 0.10 mag uncertainty due to uncertain evolutionary status.

Distances

$M_V$(RR)=0.46 +0.13 –0.09 mag
Note this depends on metallicity!

**Table 2.** Monte Carlo input parameters.

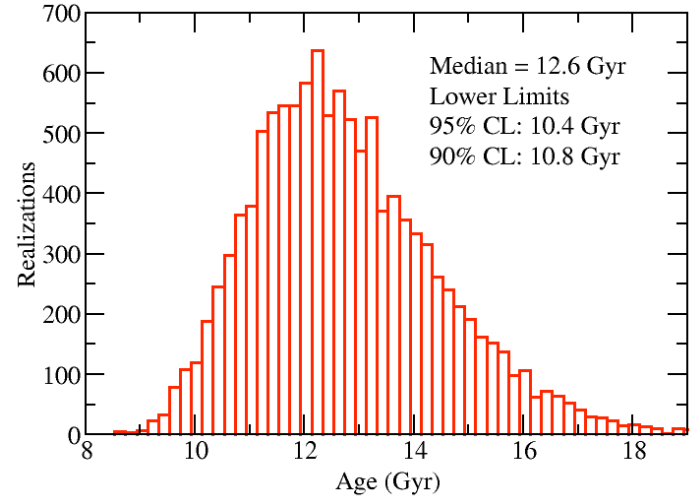
| Parameter | Distribution | References |
|---|---|---|
| [O/Fe] | 0.45 ± 0.25 (uniform) | (*37–41*) |
| Convection: Mixing length | 1.85 ± 0.25 (uniform) | (*5*) |
| Helium abundance | 0.2475 ± 0.0025 (uniform) | (*30, 42, 43*) |
| $^{14}N + p \rightarrow {}^{15}O + \gamma$ reaction rate | 1.00 ± 0.12 (uniform) | (*5*) |
| Helium diffusion | 0.50 ± 0.30 (uniform) | (*44–47*) |
| Color transformation | Binary | (*48, 49*) |
| Opacities below $10^4$ K | 1.0 ± 0.3 (uniform) | (*5*) |

Stellar models

Errors in estimates of GC ages (Krauss & Chaboyer 2003)

Globular Cluster Ages Error Budget

| Parameter | Error +/- % |
|---|---|
| $M_v(RR)$(distance) | 16 |
| [ O/*Fe*] | 7 |
| Treatment of convection | 5 |
| Helium diffusion | 4 |
| $^{14}N \rightarrow {}^{15}O$ rate | 3 |
| Color transformation | 3 |




Best fit = 13.2 +/- 1.5 Gyr

Range of possible GC ages (Chaboyer & Krauss 2003)

# Recent Results for GC's

- So the globular clusters are somewhere between ~11.7 and 14.7 Gyr old
- Since it probably takes 1-2 Gyr for galaxies to form, we need to add that to the GC ages to get the age of the universe
- Note that older estimates were closer to 13-17 Gyr old, what changed?
  - Distances to globular clusters increased by ~10% based on the Hipparcos calibration of the absolute magnitudes of subdwarfs (lowers ages by ~20%)
  - Inputs to stellar evolutionary models
  - Younger ages more compatible with ages estimated from expansion of universe (phew!)