Cluster X-ray Gas

Galaxies only contain about 20% of the baryons present in rich clusters. The other $\sim$ 80% resides in the intracluster medium. These baryons can find their way into the intracluster medium in 3 ways:

They can fall into the cluster as it forms
They can be blown out of small galaxies by winds
They can be swept out of larger galaxies by ram pressure
They can be ejected from galaxies by the jets and winds of AGN

Let's consider the first three of these mechanisms one at a time. The first is simple. A rich cluster contains $\mathcal{M} \sim 10^{15}\mathcal{M}_\odot$ of material within a ~ 1 Mpc radius. If an atom of mass m_H falls into this cluster from infinity, then its thermalization temperature is given by

$$\frac{G\mathcal{M}m_H}{R} \sim \frac{1}{2}m_H v^2 \sim \frac{3}{2}kT \Longrightarrow \ T \sim 10^8 \text{ K} \tag{10.01}$$

In the absense of cooling, this gas will emit x-rays.

Galactic Winds

[Mathews & Baker 1971, *Ap.J.,* **170,** 241]

Consider an elliptical (or spheroidal) galaxy of luminosity, $\mathcal{L}$, and mass-to-light ratio, Υ. The stars in the galaxy are losing mass at the normal rate of $\dot{\mathcal{M}}_{ISM} \sim 10^{-11}\mathcal{M}_\odot$ per year per unit solar luminosity of the galaxy. Because the stars are moving isotropically within this galaxy, the gas will thermalize at a temperature given by

$$\frac{G\Upsilon\mathcal{L}m_H}{R} \sim \frac{1}{2}m_H v^2 \sim \frac{3}{2}kT_{ISM} \tag{10.02}$$

In other words,

$$T_{ISM} \sim \frac{1}{3}\frac{m_H}{k}\frac{G\Upsilon\mathcal{L}}{R} \tag{10.03}$$

Now assume that supernovae occasionally go off in the galaxy, and eject $\dot{\mathcal{M}}_{SN}$ per year per unit solar luminosity into the ISM/null. This matter will likely be thrown off at $v_{SN} \sim 10,000$ km s^{-1} and thermalize with an efficiency, ϵ. (In other words, $1-\epsilon$ of the bulk motion energy of the ejecta will be radiated away.) The temperature of this ejecta, if thermalized, would be

$$\epsilon \cdot \frac{1}{2}m_H v_{SN}^2 \sim \frac{3}{2}kT_{SN} \tag{10.04}$$

or

$$T_{SN} = \epsilon \cdot \frac{1}{3}\frac{m_H}{k}v_{SN}^2 \tag{10.05}$$

When this (much hotter) material mixes with the ambient ISM, it heats it. The final temperature of the system will become

$$T_{new} = \frac{\dot{\mathcal{M}}_{ISM}T_{ISM} + \dot{\mathcal{M}}_{SN}T_{SN}}{\dot{\mathcal{M}}_{ISM} + \dot{\mathcal{M}}_{SN}} \tag{10.06}$$

Explicitly, this is

$$T_{new} = \frac{1}{3}\frac{m_H}{k}\frac{\dot{\mathcal{M}}_{ISM}\frac{G\Upsilon\mathcal{L}}{R} + \dot{\mathcal{M}}_{SN}\,\epsilon\, v_{SN}^2}{\dot{\mathcal{M}}_{ISM} + \dot{\mathcal{M}}_{SN}} \tag{10.07}$$

Next, we consider the escape velocity for the system,

$$v_{esc} = \sqrt{\frac{2G\mathcal{M}}{R}} \tag{10.08}$$

which means that the escape temperature is given by

$$\frac{1}{2}m_H v_{esc}^2 = \frac{3}{2}kT_{esc} \Longrightarrow T_{esc} = \frac{2}{3}\frac{m_H}{k}\frac{G\Upsilon\mathcal{L}}{R} \tag{10.09}$$

Now we ask the question: when is the ISM temperature greater than the escape temperature?

$$\frac{1}{3}\frac{m_H}{k}\frac{\dot{\mathcal{M}}_{ISM}\frac{G\Upsilon\mathcal{L}}{R} + \dot{\mathcal{M}}_{SN}\,\epsilon\, v_{SN}^2}{\dot{\mathcal{M}}_{ISM} + \dot{\mathcal{M}}_{SN}} > \frac{2}{3}\frac{m_H}{k}\frac{G\Upsilon\mathcal{L}}{R} \tag{10.10}$$

which, with a little math, becomes

$$\mathcal{L}_{wind} < \frac{\epsilon\, v_{SN}^2 R}{G\Upsilon}\frac{\dot{\mathcal{M}}_{SN}}{\dot{\mathcal{M}}_{ISM} + 2\dot{\mathcal{M}}_{SN}} \tag{10.11}$$

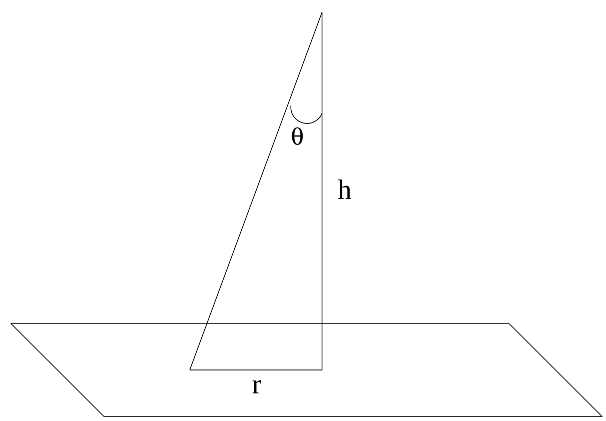
For reasonable numbers, such as $\epsilon \sim 0.1$, $\dot{\mathcal{M}}_{ISM} \sim 10^{-11}\mathcal{M}_{\odot}$, $\dot{\mathcal{M}}_{SN} \sim 10^{-13}\mathcal{M}_{\odot}$ (both per year per solar luminosity), $\Upsilon \sim 10$, and $R \sim 10$ kpc, $\mathcal{L}_{wind} \sim 2 \times 10^{10}\mathcal{L}_{\odot}$. Small galaxies therefore cannot hold onto their interstellar medium. Once a supernova goes off, it will heat the ISM, and create a galactic wind (sometimes called a Mathews-Baker wind).

Galaxy Sweeping

[Gunn & Gott 1972, *Ap.J.*, **176,** 1]

Interstellar matter may be swept out of a galaxy by ram pressure, as the classic calculation of Gunn & Gott shows. To repeat their analysis, consider a spiral galaxy moving through a cluster that already has a low-density x-ray gas. Also, following their prescription, assume that most of the mass of the spiral resides in its disk. (This is not true, of course, but since the paper was written before the discovery of dark matter halos, we'll go along with it.)

Now let's approximate a spiral galaxy as an infinite plane, with surface mass density σ_*. From freshman physics, the force required to lift a particle of mass m from the surface of the galaxy to infinity is



$$\frac{F}{m} = \int_0^\infty G \cdot \frac{2\pi r \sigma_*}{(r^2+h^2)} \cos\theta \, dr = \int_0^\infty \frac{2\pi r \sigma_* G}{(r^2+h^2)} \cdot \frac{h}{(r^2+h^2)^{1/2}} \, dr$$

$$= \int_0^\infty 2\pi G \sigma_* h \cdot \frac{r}{(r^2+h^2)^{3/2}} dr = 2\pi G \sigma_* \qquad (10.12)$$

Now let the surface density of gas in the plane of the spiral galaxy be σ_{gas}. Since the mass of gas contained within a surface area dA is related to the surface density by $dm = \sigma_{gas} dA$, the pressure required to strip gas from the galaxy is

$$P = F \cdot \frac{dm}{dA} = 2\pi G \sigma_* \sigma_{gas} \qquad (10.13)$$

Now when a galaxy moves through a cluster, the gas in the disk will feel a ram pressure due to x-ray gas. If the density of the x-ray gas is ρ_x, then, from fluid mechanics, the ram pressure is

$$P = \rho_x v^2 \tag{10.14}$$

Consequently, the gas inside the galaxy will get stripped out by the ram pressure when

$$\rho_x v^2 > 2\pi G \sigma_* \sigma_{gas} \tag{10.15}$$

This is the Gunn-Gott criterion for sweeping of gas from spirals. When does this occur? Well, there are 2×10^{11} stars in the Milky Way, and the size of the Milky Ways disk is $R \sim 20$ kpc. If a typical Milky Way star has one solar mass, then the approximate stellar surface density is $\sigma_* \sim 2 \times 10^{11}/\pi R^2 \sim 0.03$ gm cm^{-2}. Also, if the density of particles in the local interstellar medium is ~ 1 atom cm^{-3} and the thickness of the disk is ~ 200 pc, then the surface density of gas is $\sigma_{gas} \sim 6 \times 10^{20}$ atoms cm^{-2}. Plugging these numbers into (10.15), and picking a typical velocity for a galaxy moving through a cluster ($v \sim 1300$ km s^{-1}), yields a requisite x-ray gas density of $\sim 5 \times 10^{-4}$ atoms cm^{-3}.

This calculation underestimates (by up to an order of magnitude), the density needed strip a galaxy, since it ignores the presence of dark matter. However, the qualitative result is the correct: it does not take much of an intercluster medium to strip spiral galaxies of their gas. When this occurs, star formation in the spirals will rapidly come to a halt (since they'll be no gas to form stars), and the galaxy will (presumably) change into a lenticular. At the same time, the increased amount of intracluster material will cause the next spiral galaxy to be stripped that much easier. As time goes on, the amount (and x-ray luminosity) of the intracluster medium can be increased substantially.

The X-ray Gas

What will be the state of the x-ray gas? Given that the galaxy velocity dispersion in a cluster is ~ 2000 km s^{-1}, a typical virial temperature of the gas will be $T \sim 10^8$ K. Also, for reference, a typical density for intracluster gas is $n_e \sim 10^{-3}$ cm^{-3}.

TIMESCALE FOR EQUIPARTITION

Let us first ask how long it takes an electron (or proton) to share its energy with its surrounding and come to thermal equilbrium. It is fairly straightforward to show that the mean free path of a charged particle in a plasma is

$$\lambda = \frac{3^{3/2}\,(kT)^2}{4\pi^{1/2} n e^4 \ln\Lambda} \tag{10.16}$$

where e is the charge of the electron (or proton), n is the number density of particles and Λ is related to the Debeye length, *i.e.*, the ratio of the largest to smallest impact parameters. (The derivation of this equation is straightforward; you will undoubtably see it, or the equivalent for a gravitational plasma of stars, in another class). Numerically, Λ is

$$\Lambda = 37.8 + \ln\left\{ \left(\frac{T}{10^8}\right) \left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1/2} \right\} \tag{10.17}$$

Note that this is a log quantity, hence it is virtually independent of n_e and T. When we plug in the numbers, the mean free path of an electron (or ion) is

$$\lambda_e \approx \lambda_i \approx 23\ \mathrm{kpc}\ \left(\frac{T}{10^8}\right)^2 \left(\frac{n_e}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1} \tag{10.18}$$

This is almost the length of a galaxy!

Now, a reasonable estimate of the equipartition time is

$$t_{eq} = \lambda / \langle v_{rms} \rangle = \lambda / \sqrt{\frac{3kT}{m}} \tag{10.19}$$

(You can do the computation in a lot more detail, but the result will stay the same, to a factor of a couple.) Thus, from (10.16)

$$t_{eq} = \frac{3m^{1/2}\,(kT)^{3/2}}{4\pi^{1/2} n e^4 \ln \Lambda} \tag{10.20}$$

For proton-proton interactions ($m = m_p$), this works out to be

$$t_{eq} = 1.4 \times 10^7 \text{ yr} \left(\frac{T_p}{10^8}\right)^{3/2} \left(\frac{n_p}{10^{-3}\text{ cm}^{-3}}\right)^{-1} \tag{10.21}$$

while for electron-electron interactions ($m = m_e$)

$$t_{eq} = 3.3 \times 10^5 \text{ yr} \left(\frac{T_e}{10^8}\right)^{3/2} \left(\frac{n_e}{10^{-3}\text{ cm}^{-3}}\right)^{-1} \tag{10.22}$$

Note that these timescales are very short (cosmologically speaking). Consequently, any new matter introduced into the intracluster medium will shortly come into equipartition with that medium.

TIMESCALE FOR COOLING

The next question to ask concerns the cooling timescale for the x-ray gas. In other words, how long with it take the temperature of the gas to decrease by $1/e$.

$$t_{cool} = T \bigg/ \frac{dT}{dt} = \left(\frac{d \ln T}{dt} \right)^{-1} \tag{10.23}$$

Because most of the ions in an x-ray gas are either hydrogen- or helium-like, there are no low-lying levels to enable collisional cooling to work. Thus, the principal cooling mechanism is thermal Bremstrahlung. Recall that energy emitted by free-free emission is

$$\begin{aligned}
\epsilon_{ff} &= \frac{32\pi e^6}{3m_e c^3} \left(\frac{2\pi}{3m_e kT} \right)^{1/2} Z^2 n_i n_e \int_0^\infty g_{ff} e^{-h\nu/kT} d\nu \\
&= \frac{32\pi e^6}{3h m_c c^3} \left(\frac{2\pi k}{3m_e} \right)^{1/2} \bar{g}\, T^{1/2} n_e \sum_i Z_i^2 n_i \\
&\approx 3 \times 10^{-27}\, n_p^2\, T^{1/2} \text{ ergs cm}^{-3} \text{ s}^{-1} \qquad (10.24)
\end{aligned}$$

Thus,

$$\epsilon_{ff} = \frac{dE}{dt} = \frac{3}{2} nk \frac{dT}{dt} = 3 \times 10^{-27}\, n_p^2\, T^{1/2} \text{ ergs cm}^{-3} \text{ s}^{-1} \tag{10.25}$$

which means that

$$t_{cool} = \frac{kT^{1/2}}{2 \times 10^{-27} n} = 2.2 \times 10^{10} \text{ yr} \left(\frac{T}{10^8} \right)^{1/2} \left(\frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \tag{10.26}$$

This is more than a Hubble time. So, unless we're talking about gas in the very center of the cluster (where the density may be high), the x-ray gas will not be cooling significantly.

TIMESCALE FOR PRESSURE WAVE PROPAGATION

Finally, let's ask how long it takes a pressure wave to cross the cluster (and distribution any inhomogeneity over the entire system). This number is simply the size of the cluster divided by the sound speed, *i.e.*,

$$t_p = D/c_s = D \bigg/ \sqrt{\frac{\gamma k T}{\mu m_H}} \tag{10.27}$$

where γ is the ratio of specific heats ($\gamma = 5/3$) and μ_H is the mean molecular weight (of the order of one). Plugging in the numbers gives

$$t_p = 6.5 \times 10^8 \text{ yr} \left(\frac{T}{10^8} \right)^{-1/2} \left(\frac{D}{\text{Mpc}} \right) \tag{10.28}$$

Once again, this is short compared to a Hubble time. Thus, the gas will be in pressure equilibrium with its surroundings. In other words, hydrostatic equilibrium will hold.

X-ray Gas and Hydrostatic Equilibrium

[Fabricant *et al.* 1980, *Ap.J.*, **241,** 552]

Given that an x-ray gas is in thermal and pressure equilibrium, it is fairly straightforward to use the distribution of gas to derive the mass distribution of the cluster. Begin with the equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{G\mathcal{M}(r)}{r^2}\rho \tag{10.29}$$

and the ideal gas equation

$$P = \frac{\rho}{\mu m_H} kT \tag{10.30}$$

If you take the derivative of (10.30) with respect to r, then

$$\frac{dP}{dr} = \frac{k}{m_H}\left\{T\frac{d\rho}{dr} + \rho\frac{dT}{dr}\right\} = -\frac{G\mathcal{M}(r)}{r^2}\rho \tag{10.31}$$

Now if we multiply through by r/ρ, and take T outside the bracket, then

$$\begin{aligned}\frac{dP}{dr} &= \frac{kT}{\mu m_H}\left\{\frac{r}{\rho}\frac{d\rho}{dr} + \frac{r}{T}\frac{dT}{dr}\right\} = -\frac{G\mathcal{M}(r)}{r} \\ &= \frac{kT(r)}{\mu m_H G}\left\{\frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r}\right\} = -\frac{\mathcal{M}(r)}{r} \qquad (10.32)\end{aligned}$$

So

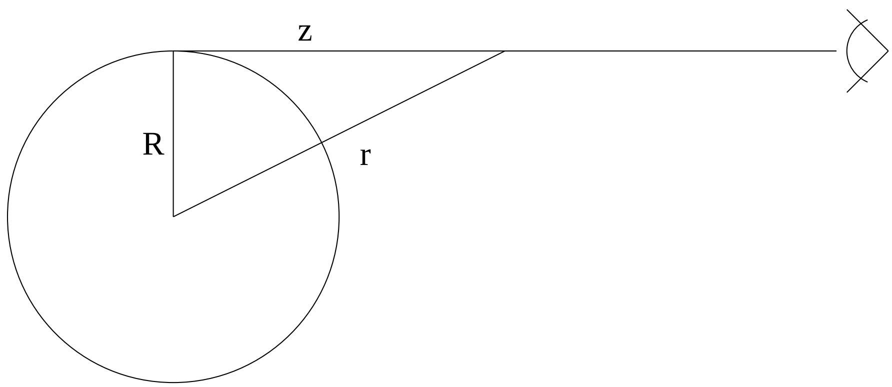
$$\mathcal{M}(r) = -\frac{kT(r)}{\mu m_H G}\left\{\frac{d\ln\rho}{d\ln r} + \frac{d\ln T}{d\ln r}\right\} r \tag{10.33}$$

Note that if the gas is truly isothermal (due to the short equipartition timescale), the last term in the bracket disappears, making the mass a function of only one derivative.

Converting from Projected Radius to True Radius

Unfortunately, equation (10.33) is of limited use. In particular, the density, ρ, is not an observable quantity; what is measured is the projected x-ray emissivity, $I_\nu(R)$, on the plane of the sky. However, it is possible go from projected intensity to three-dimensional emissivity, $\epsilon_\nu(r)$ via an Abel integral.

If the emitting x-ray plasma is spherically symmetric, then the flux one observes at any projected position in the cluster is



$$I_\nu(R) = 2\int_R^\infty \epsilon_\nu(r)dz = 2\int_R^\infty \frac{\epsilon_\nu(r)r}{(r^2 - R^2)^{1/2}}dr \qquad (10.34)$$

Now let's introduce a dummy variable ξ (which takes the place of r), and multiple each side of the equation so that

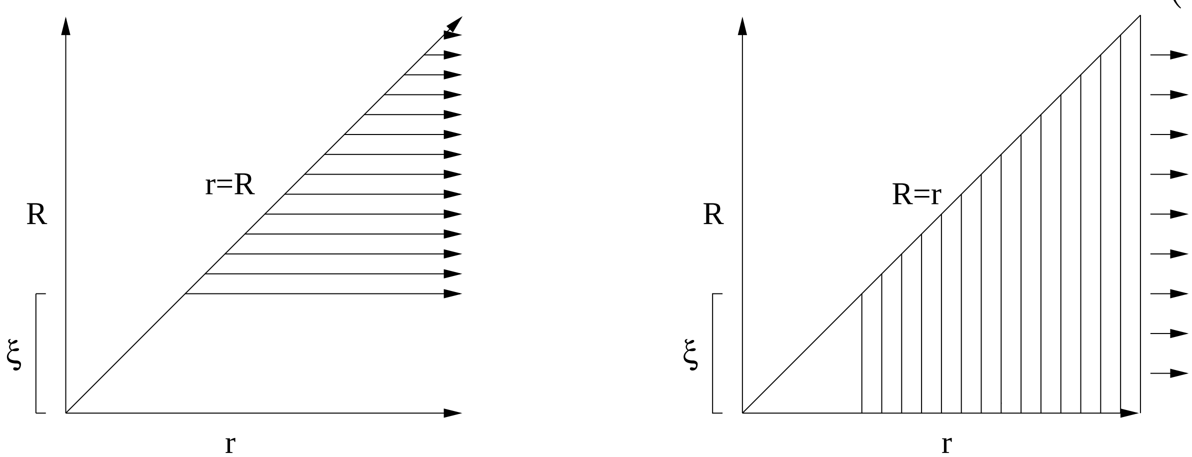
$$I_\nu(R)\frac{RdR}{(R^2 - \xi^2)^{1/2}} = 2\int_R^\infty \frac{\epsilon_\nu(r)rdr}{(r^2 - R^2)^{1/2}}\frac{RdR}{(R^2 - \xi^2)^{1/2}} \qquad (10.35)$$

Now let's integrate both sides over dR from ξ to infinity

$$\int_\xi^\infty \frac{I_\nu(R)RdR}{(R^2 - \xi^2)^{1/2}} = 2\int_\xi^\infty RdR\int_R^\infty \frac{\epsilon_\nu(r)rdr}{(r^2 - R^2)^{1/2}(R^2 - \xi^2)^{1/2}} \qquad (10.36)$$

Now comes the tricky part. Let's switch the order of the integration. As the figure below shows, in order to do this, we must also change the limits of the integration. In the first case, r is integrated from R to infinity (parallel to the x-axis) and R is integrated from ξ to infinity. In the second case, to cover the same area, R goes from ξ to r, and R is taken from ξ to infinity. Thus

$$\int_{\xi}^{\infty} \frac{I_\nu(R)RdR}{(R^2-\xi^2)^{1/2}} = 2\int_{\xi}^{\infty} \epsilon_\nu r dr \int_{\xi}^{r} \frac{RdR}{(r^2-R^2)^{1/2}(R^2-\xi^2)^{1/2}} \tag{10.37}$$



Now the second integral on the right hand side can be evaluated. If we substitute

$$\eta^2 = \frac{R^2-\xi^2}{r^2-\xi^2} \tag{10.38}$$

For the integral is over R, r is held constant, so

$$\int_{\xi}^{r} \frac{RdR}{(r^2-R^2)^{1/2}(R^2-\xi^2)^{1/2}} = \int_0^1 \frac{(r^2-\xi^2)\eta d\eta}{(r^2-\xi^2)^{1/2}(1-\eta^2)^{1/2}(r^2-\xi^2)^{1/2}\eta}$$

$$= \int_0^1 \frac{d\eta}{(1-\eta^2)^{1/2}} = \frac{\pi}{2} \tag{10.39}$$

So

$$\int_{\xi}^{\infty} \frac{I_\nu(R)RdR}{(R^2-\xi^2)^{1/2}} = \pi \int_{\xi}^{\infty} \epsilon_\nu(r) r dr \tag{10.40}$$

or, if we differentiate,

$$\epsilon_\nu(\xi) = -\frac{1}{\pi\xi}\frac{d}{d\xi}\int_{\xi}^{\infty} I_\nu(R)\frac{RdR}{(R^2-\xi^2)^{1/2}} \tag{10.41}$$

Converting from Density to Flux

The projected x-ray flux distribution can be converted to the true x-ray flux distribution via the Abel integral. To go from flux to density, consider that the x-ray emissivity from thermal bremsstrahlung is

$$\epsilon_{ff} = \frac{32\pi e^6}{3m_e c^3}\left(\frac{2\pi}{3m_e kT}\right)^{1/2} Z^2 n_e n_i \int_0^\infty g_{ff} e^{-h\nu/kT} d\nu = n_e^2 \Lambda(T) \tag{10.42}$$

Consequently,

$$\langle \epsilon_{ff} \rangle = \langle n_e^2 \Lambda(T) \rangle = \rho^2 \langle \Lambda(T) \rangle \tag{10.43}$$

where the angle brackets represent an integral over a energy bandpass (from ν_1 to ν_2). Thus

$$\frac{d\ln\langle \epsilon_{ff}\rangle}{d\ln r} = 2\frac{d\ln\rho}{d\ln r} + \frac{d\ln\langle\Lambda(T)\rangle}{d\ln r} \tag{10.44}$$

Now we solve for ρ and re-write the derivative of $\Lambda(T)$, so

$$\frac{d\ln\rho}{d\ln r} = \frac{1}{2}\frac{d\langle \epsilon_{ff}\rangle}{d\ln r} - \frac{1}{2}\frac{d\ln\langle\Lambda(T)\rangle}{d\ln T}\frac{d\ln T}{d\ln r} \tag{10.45}$$

If we then substitute this in (10.33) we get

$$\mathcal{M}(r) = -\frac{kTr}{\mu m_H G}\left\{\frac{d\ln T}{d\ln r} + \frac{1}{2}\frac{d\ln\langle\epsilon_{ff}\rangle}{d\ln r} - \frac{1}{2}\frac{d\ln\langle\Lambda(T)\rangle}{d\ln T}\cdot\frac{d\ln T}{d\ln r}\right\} \tag{10.46}$$

or

$$-\frac{\mu m_H G \mathcal{M}(r)}{kTr} - \frac{1}{2}\frac{d\ln\langle\epsilon_{ff}\rangle}{d\ln r} = \frac{d\ln T}{d\ln r}\left\{1 - \frac{1}{2}\frac{d\ln\langle\Lambda(T)\rangle}{d\ln T}\right\} \tag{10.47}$$

Note the terms in the above equation. The derivative of $\Lambda(T)$ details how the free-free emission in an x-ray band changes with temperature; it is well known from basic physics via (10.42). Also known is the derivative of the x-ray emission with radius; this comes from the Abel inversion of the observed surface brightness. Therefore, given a function for $T(r)$ and a boundary condition (usually the central temperature or pressure at infinity), this ordinary differential equation can be solved for mass.

Isothermal X-ray Gas

If an x-ray gas is in hydrostatic equilibrium, then

$$\frac{dP}{dr} = -\frac{G\mathcal{M}(r)}{r^2}\rho = \frac{d\Omega}{dr}\rho \tag{10.48}$$

where Ω is the local potential. In addition, if we take the derivative of the ideal gas law,

$$\frac{dP}{dr} = \frac{kT}{\mu m_H}\frac{d\rho}{dr} \tag{10.49}$$

Putting these two equations together yield

$$\frac{kT}{\mu m_H}\frac{d\ln\rho}{dr} = -\frac{d\Omega}{dr} \tag{10.50}$$

Now consider: the x-ray gas isn't the only thing effected by the cluster potential. The galaxies are also moving about. Let's define a characteristic "temperature" of the galaxies, $T_{\rm gal}$, via

$$\frac{1}{2}\mu m_H\langle v^2\rangle = \frac{3}{2}kT_{\rm gal} \tag{10.51}$$

where $\langle v^2\rangle$ is the mean square velocity of the galaxies. Note, however, that this is *not* the observed velocity dispersion: since we only measure radial velocities, all we can see is one component of the motion. The observed velocity dispersion, $\sigma_{\rm gal}$, is related to the true dispersion by $\langle v^2\rangle = 3\sigma_{\rm gal}$. (Of course, this is only true if the orbits are isotropic ($\langle v_x\rangle = \langle v_y\rangle = \langle v_z\rangle$) and the cluster is virialized.) So, from (10.51)

$$\frac{1}{2}\mu m_H 3\sigma_{\rm gal}^2 = \frac{3}{2}kT_{\rm gal} \quad\Longrightarrow\quad T_{\rm gal} = \frac{\mu m_H\sigma_{\rm gal}^2}{k} \tag{10.52}$$

If the galaxies are in virial equilibrium, then this temperature is related to the potential by

$$2K + \Omega = 0 \quad \Longrightarrow \quad \sigma_{\rm gal}^2 = \frac{\Omega}{3} \quad \Longrightarrow \quad T_{\rm gal} = \frac{\mu m_H}{3k}\Omega \qquad (10.53)$$

But how does this temperature compare to that of the x-ray gas. One can imagine that the x-ray gas may be cooler than the galaxies, due to its Bremstralung radiation, or the inclusion of non-virialized, possibly foreground or background galaxies in the velocity dispersion measurement. Or, one can hypothesize that the gas is hotter, due to infall. (Recall that the kinematic energy an object has from the virial theorum is half that of a particle dropped from infinity.)

To parameterize the problem, let's define β as the ratio of the virial temperature of the galaxies to the x-ray temperature of the gas, *i.e.*,

$$\beta = \frac{\mu m_H}{kT}\sigma_{\rm gal}^2 \qquad (10.54)$$

Since both the gas and the galaxies must feel the same potential, then, from (10.50) and (10.52)

$$\frac{kT}{\mu m_H}\frac{d\ln\rho_x}{dr} = \sigma_{\rm gal}^2\frac{d\ln\rho_{\rm gal}}{dr} = -\frac{d\Omega}{dr} \qquad (10.55)$$

where ρ_x is the density of the x-ray gas, and $\rho_{\rm gal}$ the density of the galaxies. Re-writing this, we get

$$\frac{d\ln\rho_x}{dr} - \frac{\mu m_H}{kT}\sigma_{\rm gal}^2\frac{d\ln\rho_{\rm gal}}{dr} = \beta\frac{d\ln\rho_{\rm gal}}{dr} \qquad (10.56)$$

If we trivially integrate over r, then

$$\rho_x = \rho_{\rm gal}^{\beta} \qquad (10.57)$$

The Total Mass of an X-ray Cluster

If the gas is isothermal, then its radial distribution should be that of an isothermal sphere. In that case, a reasonable approximation for the density distribution over the inner ~ 2 core radii is

$$\rho(r) = \frac{\rho_0}{(1+\tilde{r}^2)^{3/2}} \tag{10.58}$$

where ρ_0 is the central density, $\tilde{r}$ is r/r_0, and r_0 is the core radius. This is sometimes called a modified Hubble Law.

The modified Hubble Law departs from an isothermal sphere at large radii. Nevertheless, it is common to extrapolate this law to infinite radius, and derive the total x-ray gas mass of the cluster in terms of the core radius and central density. (But note: this only works if $\beta > 1$; otherwise, the mass is infinite, even with this approximation.) The calculation is fairly straightforward. One can measure r_0 by counting galaxies and fitting the radial distribution to an isothermal profile (or a Hubble law). One can also measure the central density from the observed amount of x-ray emission. So

$$\begin{aligned}\mathcal{M}_x &= \int_0^\infty 4\pi r^2 \frac{\rho_0}{\left(1+(r/r_0)^2\right)^{3\beta/2}} dr \\ &= 4\pi\rho_0 r_0^3 \int_0^\infty \tilde{r}^2 \left(1+\tilde{r}^2\right)^{-3\beta/2} d\tilde{r} \end{aligned} \tag{10.59}$$

If we let $x = \tilde{r}^2$, then

$$\begin{aligned}\mathcal{M}_x &= 4\pi\rho_0 r_0^3 \int_0^\infty \frac{1}{2} x^{1/2} \left(1+x\right)^{-3\beta/2} dx \\ &= 2\pi\rho_0 r_0^3 \frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{3(\beta-1)}{2}\right)}{\Gamma\left(\frac{3\beta}{2}\right)}\end{aligned}$$

$$= \pi^{3/2} \rho_0 r_0^3 \frac{\Gamma\left(\frac{3(\beta-1)}{2}\right)}{\Gamma\left(\frac{3\beta}{2}\right)} \tag{10.60}$$

or

$$\mathcal{M}_x = 3.15 \times 10^{12} \mathcal{M}_\odot \left(\frac{n_0}{10^{-3}\ \mathrm{cm}^{-3}}\right) \left(\frac{r_0}{0.25\ \mathrm{Mpc}}\right) \frac{\Gamma\left(\frac{3(\beta-1)}{2}\right)}{\Gamma\left(\frac{3\beta}{2}\right)} \tag{10.61}$$

where Γ is the gamma function.

β and the Flux Distribution of an X-ray Cluster

One can determine β by directly measuring the temperature of the intracluster medium and comparing it to the velocity dispersion measured for the galaxies. However, an alternative method, which is often used is to integrate (10.34) over the line-of-sight, and compare the result to the observed x-ray surface brightness. From (10.34) this surface brightness should be

$$I(R) = \int_R^\infty n_e n_i \Lambda(T) dz = 2\Lambda(T) \left(\frac{n_e}{n_i}\right) \int_R^\infty \frac{n_i^2(r) r}{(r^2 - R^2)^{1/2}} dr \tag{10.62}$$

If we substitute for n_i^2 using (10.57) and (10.58), and let $x = (r^2 - R^2)/(r_0^2 + R^2)$, then

$$\begin{aligned} I(R) &= 2n_0 \Lambda(T) \left(\frac{n_e}{n_i}\right) \int_R^\infty \frac{r dr}{(r^2 - R^2)^{1/2} \left(1 + (r^2/r_0^2)\right)^{3\beta}} \\ &= 2n_0^2 r_0^{6\beta} \left(\frac{n_e}{n_i}\right) \int_R^\infty \frac{r dr}{(r^2 - R^2)^{1/2} (r_0^2 + r^2)^{3\beta}} \\ &= 2n_0^2 r_0^{6\beta} \left(\frac{n_e}{n_i}\right) \frac{1}{2} (r_0 + R)^{-3\beta + \frac{1}{2}} \int_0^\infty x^{-1/2} (1+x)^{-3\beta} dx \\ &= n_0^2 r_0^{6\beta} \left(\frac{n_e}{n_i}\right) (r_0 + R)^{-3\beta + \frac{1}{2}} \frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(3\beta - \frac{1}{2}\right)}{\Gamma(3\beta)} \\ &= \sqrt{\pi} \left(\frac{n_e}{n_i}\right) n_0^2 r_0 \left\{1 + \left(\frac{r}{r_0}\right)^2\right\}^{-3\beta + \frac{1}{2}} \frac{\Gamma\left(3\beta - \frac{1}{2}\right)}{\Gamma(3\beta)} \end{aligned} \tag{10.63}$$

When one fits the x-ray surface brightness for many clusters in this manner, one finds $\langle\beta\rangle \sim 0.65$. On the other hand, when one measures the galactic velocity dispersion and compares that number to the gas temperature as determined directly by fitting x-ray spectra, then $\langle\beta\rangle \sim 1.2$. Either

a) The galaxies and/or the hot gas are not isothermal.

b) The galaxies are not in isotropic orbits.

c) The galactic velocity dispersion is overestimated by the inclusion of contaminating foreground and background galaxies.

d) The x-ray gas and galaxies do not see the same potential.

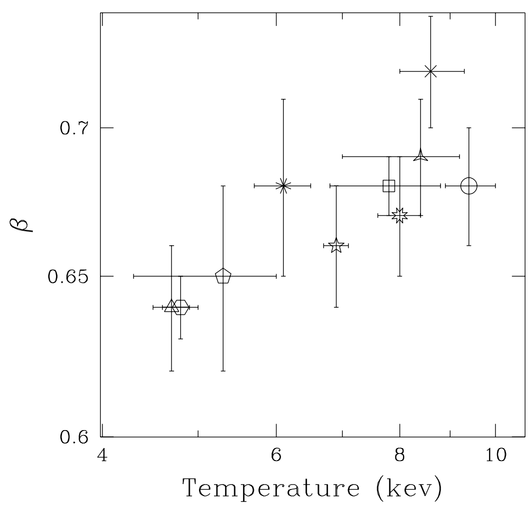


Fig. 10. β-temperature relation. β is the parameter obtained when fitting with a β−model the X-ray surface brightness profile. There is a trend to find larger β values for larger temperatures.

[Castillo-Morales & Schindler 2003]

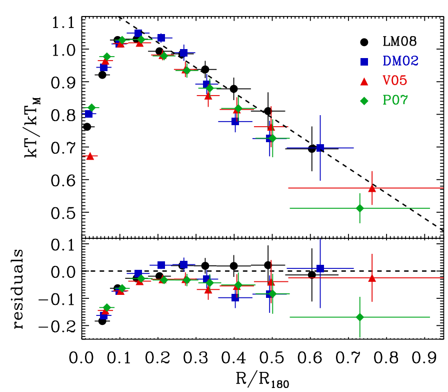


Fig. 21. Upper panel: mean temperature profiles obtained from this work (black circles, LM08), by De Grandi & Molendi (blue squares, DM02), by Vikhlinin et al. (red upward triangles, V05), and by Pratt et al. (green diamonds, P07). All profiles are rescaled by kT_M and R_{180} as defined in Sect. 4. The dashed line shows the best fit with a linear model beyond 0.2 R_{180} (see Sect. 6.1) and is drawn to guide the eye. Lower panel: residuals with respect to the linear model. The LM08 profile is the flattest one.

[Leccardi & Molendi 2008]